Space-Time Processing for Multi-User Multi-Antenna Z Channels with Quantized Feedback

Feng Li, *Member, IEEE*

Abstract— In this paper, we investigate how to send space time codes with full diversity and low decoding complexity for Z channels with quantized feedback. We assume that we have 2 transmitters and 2 receivers. Each transmitter sends code words to respective receiver at the same time. We propose a codebook design and feedback scheme that combines space-time codes and array processing to achieve low-complexity decoding and full diversity for transmitted signals. To our best knowledge, this is the first scheme with quantized feedback which can achieve low-complexity decoding and full diversity for any transmitted code word in Z channel when all the users transmit at the same time. Simulation results validate our theoretical analysis.

Index Terms— **Space-time codes, array processing, full diversity, precoder design, interference cancellation, Z channels, quantized feedback.**

I. Introduction

In wireless communications systems, space-time coding explores the utilization of multiple transmit antennas to improve the spectral efficiency and the performance over fading channels. Significant progress has been made on the research of space-time coding in recent years [1]. Later it has been applied in multiple access channels to improve the system performance [2]–[7]. Many space-time processing techniques have been used in multiple access channels to enhance system performance by canceling the interference from different users [8]–[13]. When it comes to Z channels [14], a scenario when there are two users each transmitting different code words to two receivers simultaneously, how to achieve low-complexity decoding and high performance is still an open problem.

One way is to use time division multiple access (TDMA) and let each transmitter send space-time codes to different receivers at different time slots. We can achieve symbol-by-symbol decoding and full diversity. However, in this case, the symbol rate for each user will only be one half. To avoid symbol rate loss, interference cancellation techniques based on space-time codes can be used to allow simultaneous transmission in Z channels. To the best of our knowledge, there is no scheme in literatures that can achieve full diversity, low complexity for Z channels when space-time codes are used to enhance the performance.

In this paper, we investigate how to achieve the low complexity decoding and the highest possible diversity with quantized feedback to improve the transmission quality for space-time codes in Z channels without losing symbol rate. Our idea to solve this problem is to design proper codebook, precoding and decoding schemes based on space-time coding with quantized feedback at the transmitter. The idea of combining space-time coding and precoding in multiuser systems is not new [15], [16]. We assume that our system operates under short-term power constraints, fixed codeword block length and limited delay. Under these constraints, there will always be some outage probability [17], [18]. For example, [17] shows that outage probability exisits for the block-fading channel with limited delay and block length. [18] points out that when the delay is finite, for any finite rate, as small as it may be, there is a nonzero outage probability independent of the code length. Thus, the diversity is an important tool to evaluate the system performance.

The outline of the paper follows next. Section II introduces our motivation and the Z channels we discuss in this paper. In Section III, we propose a precoding scheme to achieve interference cancellation. In Section IV, our decoding scheme is proposed. We propose the codebook and feedback design to achieve full diversity in Section V. Simulation results are presented in Section VI and Section VII concludes the paper.

II. Motivation and Channel Model

In a point-to-point MIMO system, i.e., one transmitter with N transmit antennas and one receiver with M receive antennas, one can use space-time codes to achieve symbol-by-symbol decoding and full diversity when the transmitter does not know the channel. The symbol rate is one. Let us consider a channel model as shown in Figure 1. We assume there are 2 users each with 2 transmit antennas and 2 receivers each with 2 receive antennas. Both users want to send different space-time codes to Receivers 1 and 2 on the same frequency band at the same time. As shown in Figure 1, User 1 wants to send codeword C to Receiver 1. User 2 wants to send codeword S to Receiver 2. The signals from User 2 will cause interference to User 1. When channel knowledge is not available at the transmitters, space-time codes combined with TDMA can be used to achieve symbol-by-symbol decoding and full diversity. But the symbol rate reduces to one half. A

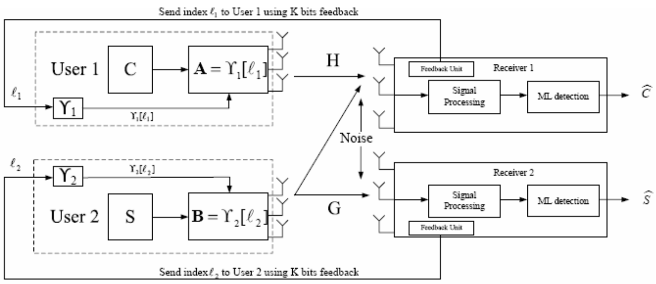

Fig. 1. Z Channel

solution to keep the symbol rate unchanged when space-time codes are used, is to combine space-time coding and array processing. In other words, we allow all transmitters to send space-time codes simultaneously to keep rate one and utilize special array processing techniques to achieve low-complexity decoding and full diversity.

In this paper. we achieve the above goals under short-term power constraints, fixed codeword block length and limited delay, when there is outage. We do not claim that our scheme can achieve capacity or full degree of freedom. After all, there is a tradeoff between diversity and multiplexing gain, which is outside the scope of this paper.

In order to achieve our goals, we propose the following scheme in time slot 1 as shown in Figure 1: First, we assume that Users 1 and 2 transmit codewords $\mathbf{C}$ and $\mathbf{S}$, respectively. And each user can receive K bits of feedback from the receiver. Second, we design a codebook Υ_1 which contains $L_1 = 2^K$ different precoding matrices for User 1 and a codebook Υ_2 which contains $L_2 = 2^K$ different precoding matrices for User 2. Each codebook is shared by its transmitter and the receiver. Also we let $\Upsilon_i[j]$ denote the jth matrix in Codebook Υ_i. Third, Receiver 1 sends back an index ℓ_1 to User 1 using K bits of feedback and Receiver 2 sends back an index ℓ_2 to User 2 using another K bits of feedback. Finally, User 1 chooses $\Upsilon_1[\ell_1]$ as its precoder $\mathbf{A}^1$ and transmits the pre-coded signals to the receiver. Also User 2 chooses $\Upsilon_2[\ell_2]$ as its precoder $\mathbf{B}^1$ and transmits the pre-coded signals to the receiver. After receiving the signals from Users 1 and 2, the receiver decodes the signals for each user separately using an array processing method.

At time slot 2, the scheme will be exactly the same as that at time slot 1. But the designed codebooks $\Upsilon_1^{'}$ for User 1 and $\Upsilon_2^{'}$ for User 2 in time slot 2 may be different from the codebooks Υ_1 and Υ_2 in time slot 1. Also the feedback indices $\ell_1^{'}$ and $\ell_2^{'}$ in time slot 2 may be different from ℓ_1 and ℓ_2 in time slot 1. As a result, the precoders $\mathbf{A}^2$ for User 1 and $\mathbf{B}^2$ for User 2 in time slot 2 may be different from $\mathbf{A}^1$ and $\mathbf{B}^1$ in time slot 1. The above is our main idea to achieve low-complexity and high performance interference-free transmission. In what follows, we will present our scheme in details.

First, let us introduce the input-output equations. We let each user transmit Alamouti Codes [20] as follows:

$$\mathbf{C} = \begin{pmatrix} c_1 & -c_2^* \\ c_2 & c_1^* \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix} \tag{1}$$

where $i, j = 1, 2$. Note that we can also use other space-time codes with rate one and Alamouti code is just one example. Let

$$\mathbf{A}^t = [a^t(i,j)]_{2\times 2}, \quad t = 1, 2 \tag{2}$$

be the precoders we need to design. They are combined with the space-time codes sent by User 1 and this is the first step of our array processing technique. Note that in order to satisfy the short-term power constraint, we need

$$||\mathbf{A}^t||_F^2 = 1 \tag{3}$$

Similarly, the precoders for User 2 is defined as

$$\mathbf{B}^t = [b^t(i,j)]_{2\times 2}, \quad t = 1, 2 \tag{4}$$

with the power constraint

$$||\mathbf{B}^t||_F^2 = 1 \tag{5}$$

The channels are quasi-static flat Rayleigh fading and keep unchanged during two time slots. Then we let

$$\mathbf{H}_l = [h_l(i,j)]_{2\times 2}, \quad l = 1, 2 \tag{6}$$

denote the channel matrix between User 1 and Receivers l, respectively. Similarly, we use

$$\mathbf{G}_l = [g_l(i,j)]_{2\times 2}, \quad l = 1, 2 \tag{7}$$

to denote the channel matrix between User 2 and Receiver l, respectively. Then the received signals at Receiver 1 at time slot t can be denoted by

$$\mathbf{y}_1^t = \mathbf{H}_1\mathbf{A}^t\mathbf{C}(t) + \mathbf{G}_1\mathbf{B}^t\mathbf{S}(t) + \mathbf{n}_1^t \tag{8}$$

where

$$\mathbf{y}_1^t = [y_1^t(i,1)]_{2\times 1}, \quad \mathbf{n}_1^t = [n_1^t(i,1)]_{2\times 1} \tag{9}$$

denote the received signals and the noise at Receiver 1, respectively, at time slot t. Similarly, at time slot t, Receiver 2 will receive the following signals

$$\mathbf{y}_2^t = \mathbf{G}_2\mathbf{B}^t\mathbf{S}(t) + \mathbf{n}_2^t \tag{10}$$

where

$$\mathbf{y}_2^t = [y_2^t(i,1)]_{2\times 1}, \quad \mathbf{n}_2^t = [n_2^t(i,1)]_{2\times 1} \tag{11}$$

Equations (8) and (10) are the channel equations on which we will base our design in this paper.

III. Precoder Design and Interference Cancellation

In this section, we will show how to design the codebook and precoders, in order to achieve interference cancellation. In Equation (8), we use

$$\mathbf{H}_{11}^t = \mathbf{H}_1\mathbf{A}^t, \mathbf{G}_{11}^t = \mathbf{G}_1\mathbf{B}^t \tag{12}$$

to denote the equivalent channel matrices. Then Equation (8) becomes

$$\mathbf{y}_1^t = \mathbf{H}_{11}^t\mathbf{C}(t) + \mathbf{G}_{11}^t\mathbf{S}(t) + \mathbf{n}_1^t \tag{13}$$

In Equation (10), if we use

$$\mathbf{G}_{21}^t = \mathbf{G}_2\mathbf{B}^t \tag{14}$$

to denote the equivalent channel matrices, we have

$$\mathbf{y}_2^t = \mathbf{G}_{21}^t\mathbf{S}(t) + \mathbf{n}_2^t \tag{15}$$

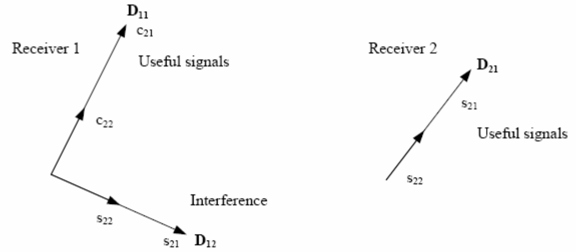

Fig. 2. Signal vector illustration at two receivers

By Equation (13), c_1, c_2, s_1, s_2 are transmitted along four equivalent channel vectors $\mathbf{H}_{11}^1(1)$, $\mathbf{H}_{11}^1(2)$, $\mathbf{G}_{11}^1(1)$, $\mathbf{G}_{11}^1(2)$, respectively. Suppose that we want to remove the signals of User 2 at Receiver 1, we can find a 2-by-1 complex vector $\mathbf{g}$ satisfying $\mathbf{g}^\dagger \mathbf{G}_{11}^1(1) = \mathbf{g}^\dagger \mathbf{G}_{11}^1(2) = 0$. Then by simply multiplying both sides of Equation (13) by $\mathbf{g}^\dagger$, we can remove the signals of User 2. This is our basic idea to achieve the interference cancellation.

However, since $\mathbf{G}_{11}^1(1)$, $\mathbf{G}_{11}^1(2)$ are 2-by-1 complex vectors, a non-zero complex vector $\mathbf{g}_{2\times 1}$ that satisfies $\mathbf{g}^\dagger \mathbf{G}_{11}^1(1) = \mathbf{g}^\dagger \mathbf{G}_{11}^1(2) = 0$ does not exist unless $\mathbf{G}_{11}^1(1) = \alpha \mathbf{G}_{11}^1(2)$, where α is a constant. Therefore, in order to cancel the interference from User 2, we need $\mathbf{G}_{11}^1(1) = \alpha \mathbf{G}_{11}^1(2)$. In other words, s_1, s_2 are transmitted in the same direction as shown in Figure 2, where $\mathbf{D}_{ij}$ is the jth direction at Receiver i. To make $\mathbf{G}_{11}^1(1) = \alpha \mathbf{G}_{11}^1(2)$, our precoders $\mathbf{A}^1$ and $\mathbf{B}^1$ should have the following properties:

$$\mathbf{A}^1(1) = \mathbf{A}^1(2), \quad \mathbf{B}^1(1) = \mathbf{B}^1(2), \tag{16}$$

Since we choose a matrix in the codebook $\mathbf{\Upsilon_1}$ as the precoder for User 1 and a matrix in the codebook $\mathbf{\Upsilon_2}$ as the precoder for User 2, Equation (16) results in:

$$\mathbf{\Upsilon_1}[\mathbf{i}](\mathbf{1}) = \mathbf{\Upsilon_1}[\mathbf{i}](\mathbf{2}), \quad \mathbf{\Upsilon_2}[\mathbf{j}](\mathbf{1}) = \mathbf{\Upsilon_2}[\mathbf{j}](\mathbf{2}), \tag{17}$$

i.e., the two columns of any matrix in codebooks $\mathbf{\Upsilon_1}$ and $\mathbf{\Upsilon_2}$ should be the same.

Similarly, in time slot 2, our precoders should satisfy

$$\mathbf{A}^2(1) = \mathbf{A}^2(2), \quad \mathbf{B}^2(1) = \mathbf{B}^2(2). \tag{18}$$

Then using the codebook $\mathbf{\Upsilon}_1^{'}$ and $\mathbf{\Upsilon}_2^{'}$, for Users 1 and 2, respectively, any matrix $\mathbf{\Upsilon}_1^{'}[\mathbf{i}]$ in the codebook $\mathbf{\Upsilon}_1^{'}$ and any matrix $\mathbf{\Upsilon}_2^{'}[\mathbf{j}]$ in the codebook $\mathbf{\Upsilon}_2^{'}$ have the following properties:

$$\mathbf{\Upsilon}_1^{'}[\mathbf{i}](\mathbf{1}) = \mathbf{\Upsilon}_1^{'}[\mathbf{i}](\mathbf{2}), \quad \mathbf{\Upsilon}_2^{'}[\mathbf{j}](\mathbf{1}) = \mathbf{\Upsilon}_2^{'}[\mathbf{j}](\mathbf{2}). \tag{19}$$

Now Equations (13) and (15) become

$$\mathbf{y}_1^t = [\mathbf{H}_{11}^t(1), \mathbf{H}_{11}^t(1)] \cdot \mathbf{C}(t) + [\mathbf{G}_{11}^t(1), \mathbf{G}_{11}^t(1)] \cdot \mathbf{S}(t) + \mathbf{n}_1^t \tag{20}$$

and

$$\mathbf{y}_2^t = [\mathbf{G}_{21}^t(1), \mathbf{G}_{21}^t(1)] \cdot \mathbf{S}(t) + \mathbf{n}_2^t \tag{21}$$

where $\mathbf{H}_{11}^t(1)$, $\mathbf{G}_{11}^t(1)$, $\mathbf{H}_{21}^t(1)$, $\mathbf{G}_{21}^t(1)$ denote the first column of matrix $\mathbf{H}_{11}^t$, $\mathbf{G}_{11}^t$, $\mathbf{H}_{21}^t$, $\mathbf{G}_{21}^t$, respectively. In order to cancel the interference from User 2, we only need to find two vectors $\mathbf{g}^1$ and $\mathbf{g}^2$ such that

$$(\mathbf{g}^1)^\dagger \cdot (\mathbf{G}_{11}^1(1)) = 0 \tag{22}$$

$$(\mathbf{g}^2)^\dagger \cdot (\mathbf{G}_{11}^2(1)) = 0 \tag{23}$$

Then we can cancel the interference from User 2, by multiplying both sides of Equation (20) with vector $(\mathbf{g}^1)^\dagger$. The detailed decoding is illustrated in the next section.

IV. Decoding with Low Complexity

In this section, we will show how to decode and analyze the decoding complexity. We first consider the decoding at receiver one. Let

$$\mathbf{g}^1 = \begin{pmatrix} (\mathbf{G}_{11}^1(2,1))^* \\ -(\mathbf{G}_{11}^1(1,1))^* \end{pmatrix}, \quad \mathbf{g}^2 = \begin{pmatrix} (\mathbf{G}_{11}^2(2,1))^* \\ -(\mathbf{G}_{11}^2(1,1))^* \end{pmatrix} \tag{24}$$

Note that at receiver one, c_1, c_2 are the desired signal and s_1, s_2 are the interference. We can cancel the interference by multiplying both sides of Equation (20) by matrix $(\mathbf{g}^t)^\dagger$. Then we get

$$(\mathbf{g}^t)^\dagger \cdot \mathbf{y}_1^t = (\mathbf{g}^t)^\dagger \mathbf{H}_{11}^t \mathbf{C}(t) + (\mathbf{g}^t)^\dagger \mathbf{n}_1^t \tag{25}$$

Here we have canceled the interference because $(\mathbf{g}^t)^\dagger \mathbf{G}_{11}^t = 0$. In order to decode the symbols, we first multiply both sides of Equations (25) by matrix $|\mathbf{g}^t|^{-1}$ to whiten the noise, i.e.,

$$\frac{(\mathbf{g}^t)^\dagger}{|\mathbf{g}^t|} \cdot \mathbf{y}_1^t = \frac{(\mathbf{g}^t)^\dagger}{|\mathbf{g}^t|} \mathbf{H}_{11}^t \mathbf{C}(t) + \frac{(\mathbf{g}^t)^\dagger}{|\mathbf{g}^t|} \mathbf{n}_1^t \tag{26}$$

After combining channel equations in two time slots, we have

$$\begin{pmatrix} \frac{(\mathbf{g}^1)^\dagger}{|\mathbf{g}^1|} \cdot \mathbf{y}_1^1 \\ \left(\frac{(\mathbf{g}^2)^\dagger}{|\mathbf{g}^2|} \cdot \mathbf{y}_1^2 \right)^* \end{pmatrix} = \begin{pmatrix} \frac{(\mathbf{g}^1)^\dagger}{|\mathbf{g}^1|} \mathbf{H}_{11}^1(1) & \frac{(\mathbf{g}^1)^\dagger}{|\mathbf{g}^1|} \mathbf{H}_{11}^1(1) \\ \left(\frac{(\mathbf{g}^2)^\dagger}{|\mathbf{g}^2|} \mathbf{H}_{11}^2(1) \right)^* & -\left(\frac{(\mathbf{g}^2)^\dagger}{|\mathbf{g}^2|} \mathbf{H}_{11}^2(1) \right)^* \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} \frac{(\mathbf{g}^1)^\dagger}{|\mathbf{g}^1|} \mathbf{n}_1^1 \\ \left(\frac{(\mathbf{g}^2)^\dagger}{|\mathbf{g}^2|} \mathbf{n}_1^2 \right)^* \end{pmatrix} \tag{27}$$

Let

$$\mathbf{H} = \begin{pmatrix} \frac{(\mathbf{g}^1)^\dagger}{|\mathbf{g}^1|} \mathbf{H}_{11}^1(1) & \frac{(\mathbf{g}^1)^\dagger}{|\mathbf{g}^1|} \mathbf{H}_{11}^1(1) \\ \left(\frac{(\mathbf{g}^2)^\dagger}{|\mathbf{g}^2|} \mathbf{H}_{11}^2(1) \right)^* & -\left(\frac{(\mathbf{g}^2)^\dagger}{|\mathbf{g}^2|} \mathbf{H}_{11}^2(1) \right)^* \end{pmatrix} \tag{28}$$

Note that $\mathbf{H}$ has the following Single Value Decomposition [21]

$$\mathbf{H} = \mathbf{U_H} \mathbf{\Sigma_H} \mathbf{V_H} = \mathbf{U_H} \mathbf{\Sigma_H} \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix} \tag{29}$$

where $\mathbf{U_H}$ is a complex matrix and $\mathbf{\Sigma_H}$, $\mathbf{V_H}$ are all real matrices. Then we can multiply both sides of Equation (27) by $\mathbf{U_H}^\dagger$ as follows

$$\mathbf{U_H}^\dagger \begin{pmatrix} \frac{(\mathbf{g}^1)^\dagger}{|\mathbf{g}^1|} \cdot \mathbf{y}_1^1 \\ \left(\frac{(\mathbf{g}^2)^\dagger}{|\mathbf{g}^2|} \cdot \mathbf{y}_1^2 \right)^* \end{pmatrix} =$$

$$\sqrt{E_s} \mathbf{U_H}^\dagger \begin{pmatrix} \frac{(\mathbf{g}^1)^\dagger}{|\mathbf{g}^1|} \mathbf{H}_{11}^1(1) & \frac{(\mathbf{g}^1)^\dagger}{|\mathbf{g}^1|} \mathbf{H}_{11}^1(1) \\ \left(\frac{(\mathbf{g}^2)^\dagger}{|\mathbf{g}^2|} \mathbf{H}_{11}^2(1) \right)^* & -\left(\frac{(\mathbf{g}^2)^\dagger}{|\mathbf{g}^2|} \mathbf{H}_{11}^2(1) \right)^* \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$+ \mathbf{U_H}^\dagger \begin{pmatrix} \frac{(\mathbf{g}^1)^\dagger}{|\mathbf{g}^1|} \mathbf{n}_1^1 \\ \left(\frac{(\mathbf{g}^2)^\dagger}{|\mathbf{g}^2|} \mathbf{n}_1^2 \right)^* \end{pmatrix}. \tag{30}$$

In the above equation, $\mathbf{U}_{\mathbf{H}}^{\dagger}\begin{pmatrix} \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{n}_1^1 \\ \left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{n}_1^2\right)^* \end{pmatrix}$ is still white noise and $\mathbf{U}_{\mathbf{H}}^{\dagger}\begin{pmatrix} \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{H}_{11}^1(1) & \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{H}_{11}^1(1) \\ \left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{H}_{11}^2(1)\right)^* & -\left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{H}_{11}^2(1)\right)^* \end{pmatrix}$ is real matrix. So if QAM is used, Equation (30) is equivalent to the following two equations

$$\begin{aligned}
&\text{Real}\left\{\mathbf{U}_{\mathbf{H}}^{\dagger}\begin{pmatrix} \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\cdot\mathbf{y}_1^1 \\ \left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\cdot\mathbf{y}_1^2\right)^* \end{pmatrix}\right\} = \\
&\sqrt{E_s}\mathbf{U}_{\mathbf{H}}^{\dagger}\begin{pmatrix} \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{H}_{11}^1(1) & \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{H}_{11}^1(1) \\ \left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{H}_{11}^2(1)\right)^* & -\left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{H}_{11}^2(1)\right)^* \end{pmatrix}\text{Real}\left\{\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}\right\} \\
&+\text{Real}\left\{\mathbf{U}_{\mathbf{H}}^{\dagger}\begin{pmatrix} \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{n}_1^1 \\ \left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{n}_1^2\right)^* \end{pmatrix}\right\}. \qquad (31)
\end{aligned}$$

$$\begin{aligned}
&\text{Imag}\left\{\mathbf{U}_{\mathbf{H}}^{\dagger}\begin{pmatrix} \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\cdot\mathbf{y}_1^1 \\ \left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\cdot\mathbf{y}_1^2\right)^* \end{pmatrix}\right\} = \\
&\sqrt{E_s}\mathbf{U}_{\mathbf{H}}^{\dagger}\begin{pmatrix} \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{H}_{11}^1(1) & \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{H}_{11}^1(1) \\ \left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{H}_{11}^2(1)\right)^* & -\left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{H}_{11}^2(1)\right)^* \end{pmatrix}\text{Imag}\left\{\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}\right\} \\
&+\text{Imag}\left\{\mathbf{U}_{\mathbf{H}}^{\dagger}\begin{pmatrix} \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{n}_1^1 \\ \left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{n}_1^2\right)^* \end{pmatrix}\right\}. \qquad (32)
\end{aligned}$$

Then we can use the Maximum-Likelihood method to decode the real parts and imaginary parts of c_1, c_2 separately. For example, when we detect the real parts of c_1, c_2, we have

$$\begin{aligned}
&\text{Real}\{\widehat{c}_1, \widehat{c}_2\} = \arg\min_{\text{Real}\{c_1,c_2\}} ||\text{Real}\left\{\mathbf{U}_{\mathbf{H}}^{\dagger}\begin{pmatrix} \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\cdot\mathbf{y}_1^1 \\ \left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\cdot\mathbf{y}_1^2\right)^* \end{pmatrix}\right\} \\
&-\sqrt{E_s}\mathbf{U}_{\mathbf{H}}^{\dagger}\begin{pmatrix} \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{H}_{11}^1(1) & \frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{H}_{11}^1(1) \\ \left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{H}_{11}^2(1)\right)^* & -\left(\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{H}_{11}^2(1)\right)^* \end{pmatrix} \\
&\times\text{Real}\left\{\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}\right\}||_{\text{F}}^2. \qquad (33)
\end{aligned}$$

Similarly, we can decode the imaginary parts of c_1, c_2, and the signals of User 2. Note that the decoding complexity is symbol-by-symbol.

Till now, we have presented our precoding, decoding methods, and some necessary properties needed by our codebooks to cancel interference for each user. Note that in order to achieve interference cancellation, the only properties needed by our codebooks are (17) and (19). The remaining degrees of freedom will be used to maximize diversity and coding gain as discussed in the next two sections.

V. Diversity Analysis and Feedback Design

In this section, we show how to achieve full diversity by designing feedback scheme and codebook. We only prove that at receiver 1, the diversity for c_1, c_2 from user 1 is full. The proof for s_1, s_2 at receiver two will be similar. First, the diversity is defined as

$$d = -\lim_{\rho\to\infty}\frac{\log P_e}{\log\rho} \qquad (34)$$

where ρ denotes the SNR and P_e represents the probability of error. We let $\mathbf{e} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} - \begin{pmatrix} \widehat{c}_1 \\ \widehat{c}_2 \end{pmatrix}$ denote the error vector. Here we add a rotation matrix $\mathbf{R}$ on the transmitted codewords to improve the system performance. Based on Equation (27), the pairwise error probability (PEP) for c_1, c_2 can be written as [22]

$$\begin{aligned}
P(\mathbf{c}\to\overline{\mathbf{c}}|\mathbf{H}) &= Q\left(\sqrt{\frac{\rho||(\mathbf{H}^{\dagger}\mathbf{H})^{\frac{1}{2}}\mathbf{Re}||_F^2}{4}}\right) \\
&= Q\left(\sqrt{\frac{\rho\mathbf{e}^{\dagger}\mathbf{R}^{\dagger}\mathbf{H}^{\dagger}\mathbf{H}\mathbf{R}\mathbf{e}}{4}}\right) \le \exp\left(-\frac{\rho\mathbf{e}^{\dagger}\mathbf{R}^{\dagger}\mathbf{H}^{\dagger}\mathbf{H}\mathbf{R}\mathbf{e}}{4}\right) \\
&= \exp\left(-\frac{\rho\lambda}{4}\right) \qquad (35)
\end{aligned}$$

where

$$\lambda = \left|\frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{H}_{11}^1(1)\right|_F^2|\widehat{e}_1+\widehat{e}_2|^2 + \left|\frac{(\mathbf{g}^2)^{\dagger}}{|\mathbf{g}^2|}\mathbf{H}_{11}^2(1)\right|_F^2|\widehat{e}_1-\widehat{e}_2|^2 \qquad (36)$$

and

$$\widehat{\mathbf{e}} = \begin{pmatrix} \widehat{e}_1 \\ \widehat{e}_2 \end{pmatrix} = \mathbf{Re} \qquad (37)$$

If we let θ^1 denote the angle between vector $\mathbf{g}^1$ and vector $\mathbf{H}_{11}^1(1)$ at time slot 1, i.e., $\cos(\theta^1) = <\mathbf{g}^1, \mathbf{H}_{11}^1(1)> = \frac{(\mathbf{g}^1)^{\dagger}\mathbf{H}_{11}^1(1)}{|(\mathbf{g}^1)^{\dagger}|\cdot|\mathbf{H}_{11}^1(1)|}$, then Equation (35) can be written as

$$\begin{aligned}
P(\mathbf{c}\to\overline{\mathbf{c}}|\mathbf{H}) &\le \exp\left(-\frac{\lambda\left|\frac{(\mathbf{g}^1)^{\dagger}}{|\mathbf{g}^1|}\mathbf{H}_{11}^1(1)\right|_F^2|\widehat{e}_1+\widehat{e}_2|^2}{4}\right) \\
&= \exp\left(-\frac{\lambda|\cos(\theta^1)|^2|\mathbf{H}_{11}^1(1)|_F^2|\widehat{e}_1+\widehat{e}_2|^2}{4}\right) \qquad (38)
\end{aligned}$$

Let Γ denote a matrix comprised of the first column of each matrix in the codebook Υ_1 for User 1, i.e., $\Gamma = [\Upsilon_1[1](1), \ldots, \Upsilon_1[L_1](1)]$, where $\Upsilon_1[i]$ is the ith matrix in codebook Υ_1 and L_i is the number of matrices in codebook Υ_i. In order to achieve full diversity, we choose a codebook Υ_1 such that the rank of Γ is full and propose the following feedback scheme:

Assuming each user has already got a codebook in each time slot, at time slot 1, Receiver 1 selects an index ℓ_1 within the range from 0 to $L_1 - 1$ and sends it back to User 1. The selection criterion is that with such an index ℓ_1, $|\mathbf{H}_{11}^1(1)|_F^2$ is maximized, where $|\mathbf{H}_{11}^1(1)|_F^2 = |\mathbf{H}_1\mathbf{A}^1(1)|^2$ and $\mathbf{A}^1 = \boldsymbol{\Upsilon}_1[\ell_1]$. Maximizing $|\mathbf{H}_{11}^1(1)|_F^2$ is equivalent to maximizing the received SINR for User 1. At the same time slot, the receiver 2 picks an index ℓ_2 and sends it back to User 2. The selection criterion is that with such an index ℓ_2, θ^1 is minimized. Note that $\mathbf{B}^1 = \boldsymbol{\Upsilon}_2[\ell_2]$. Similarly, in time slot 2, the receiver finds an index ℓ_2' and sends it back to User 2. The selection criterion is that with such an index ℓ_2', $|\mathbf{G}_{21}^2(1)|_F^2$ is maximized. The receiver also finds an index ℓ_1' and sends it back to User 1. The selection criterion is that with such an index ℓ_1', θ^2 is minimized, where θ^2 denote the angle between vector $\mathbf{h}^2 = \begin{pmatrix} (\mathbf{H}_{11}^2(2,1))^* \\ -(\mathbf{H}_{11}^2(1,1))^* \end{pmatrix}$ and vector $\mathbf{G}_{11}^2(1)$ at time slot 2. Now we show that by doing so, we can also achieve full diversity and maximize coding gain within our system framework.

In Equation (38), since we choose our precoder $\mathbf{A}^1$ from the codebook $\boldsymbol{\Upsilon}_1$ such that $|\mathbf{H}_{11}^1(1)|^2$ is maximized, we have

$$|\mathbf{H}_{11}^1(1)|^2 = |\mathbf{H}\mathbf{A}^1(1)|^2 \geq \frac{|\mathbf{H}\Gamma|^2}{L} \quad (39)$$

We assume Γ has the following Singular Value Decomposition

$$\Gamma = \mathbf{U}_\Gamma \boldsymbol{\Sigma}_\Gamma \mathbf{V}_\Gamma^\dagger = \mathbf{U}_\Gamma \begin{pmatrix} \lambda_1^\Gamma & 0 \\ 0 & \lambda_2^\Gamma \end{pmatrix} \mathbf{V}_\Gamma^\dagger. \quad (40)$$

Then we have

$$|\mathbf{H}\Gamma|^2 = (\lambda_1^\Gamma)^2(|h_{11}'|^2+|h_{21}'|^2)+(\lambda_2^\Gamma)^2(|h_{12}'|^2+|h_{22}'|^2) \quad (41)$$

where $\mathbf{H}\mathbf{U}_\Gamma = \begin{pmatrix} h_{11}' & h_{12}' \\ h_{21}' & h_{22}' \end{pmatrix}$ and each element is independent Gaussian random variables with mean 0 and variance 1. From Equation (38), we have

$$\begin{aligned} &P(\mathbf{c} \to \overline{\mathbf{c}}|\mathbf{H}) \\ &\leq \exp(-\frac{\lambda}{4L}|\cos(\theta^1)|^2((\lambda_1^\Gamma)^2(|h_{11}'|^2 + |h_{21}'|^2) \\ &+(\lambda_2^\Gamma)^2(|h_{12}'|^2 + |h_{22}'|^2))|\widehat{e}_1 + \widehat{e}_2|^2) \end{aligned} \quad (42)$$

So

$$\begin{aligned} &P(\mathbf{d} \to \overline{\mathbf{d}}) \leq \\ &E[\exp(-\frac{\rho}{4L}(|\cos\theta^1|^2 \cdot (|\lambda_1^\Gamma|^2(|h_{11}'|^2 + |h_{21}'|^2) \\ &+|\lambda_2^\Gamma|^2(|h_{12}'|^2 + |h_{22}'|^2)) \cdot |\widehat{e}_1 + \widehat{e}_2|^2))] \end{aligned}$$

$$\begin{aligned} &= E[E[\exp(-\frac{\rho}{4L}(|\cos\theta^1|^2 \cdot (|\lambda_1^\Gamma|^2(|h_{11}'|^2 + |h_{21}'|^2) \\ &+|\lambda_2^\Gamma|^2(|h_{12}'|^2 + |h_{22}'|^2)) \cdot |\widehat{e}_1 + \widehat{e}_2|^2))|\theta^1]] \end{aligned} \quad (43)$$

$$\leq E\left[\frac{1}{\prod_{j=1}^2[1 + (\rho|\cos\theta^1|^2|\lambda_j^\Gamma|^2|\widehat{e}_1 + \widehat{e}_2|^2/8L)]^2}\right]. \quad (44)$$

At high SNRs, one can neglect the one in the denominator and get

$$P(\mathbf{d} \to \overline{\mathbf{d}}) \leq \left(\frac{\rho}{8L}\right)^{-4} \prod_{j=1}^2(|\lambda_j^\Gamma| \cdot |\widehat{e}_1 + \widehat{e}_2|)^{-4} E\left[\frac{1}{|\cos\theta^1|^8}\right] \quad (45)$$

Since matrix Γ is full rank, $\lambda_j^\Gamma \neq 0$. So the diversity for User 1 is 4, full diversity. Similarly, we can prove that the diversity for User 2 is also full. Till now, we have shown that each matrix in the codebook should have full rank to achieve full diversity. The two columns of each matrix in the codebook should be the same to achieve interference cancellation. There are still some degrees of freedom left in the codebook design. We utilize the remaining degrees of freedom to achieve the highest coding gain within our framework. By Equation (38), in order to increase coding gain, we need to maximize $\cos(\theta^1)|^2$ and $|\mathbf{H}_{11}^1(1)|_F^2$. Since $|\mathbf{H}_{11}^1(1)|_F^2 = |\mathbf{H}_1\mathbf{A}^1(1)|^2$, it is well-known that we can choose $\mathbf{A}^1(1) = \mathbf{V}_{\mathbf{H}_1}(1)$, where $\mathbf{H}_1 = \mathbf{U}_{\mathbf{H}_1}\Sigma_{\mathbf{H}_1}\mathbf{V}_{\mathbf{H}_1}^\dagger$. $\mathbf{V}_\mathbf{H}(1)$ is the singular vector of $\mathbf{H}_{1\varphi}$ corresponding to the largest singular value and we assume $\lambda_1 > \lambda_2$ without loss of generality. If we have perfect feedback, we can simply choose $\mathbf{A}^1(1) = \frac{1}{\sqrt{2}}\mathbf{V}_{\mathbf{H}_1}(1)$ and the precoder $\mathbf{A}^1 = \frac{1}{\sqrt{2}}[\mathbf{V}_{\mathbf{H}_1}(1), \mathbf{V}_{\mathbf{H}_1}(1)]$. Since we only have access to quantized feedback, we should design a codebook in which we can find a matrix whose column is the best approximation to $\frac{1}{\sqrt{2}}\mathbf{V}_{\mathbf{H}_1}(1)$.

It has been shown in [23] that $\mathbf{V}_{\mathbf{H}_1}(1)$ is an isotropically distributed unitary vector. The intuitive meaning of an isotropically distributed complex unit vector is that it is equally likely to point in any direction in complex space. Therefore, the problem to design a codebook to maximize $|\mathbf{H}_{11}^1(1)|_F^2$ becomes how to pack one-dimensional subspaces of a complex space known as Grassmannian line packing [24]. In other words, it is the problem of finding a set of L_1 one-dimensional subspaces in the complex space that maximize the minimum distance between any pair of subspaces in the set.

The problem of finding optimal line packings using analytical or numerical methods is not new [24]–[27]. We utilize the existing methodologies in the literature to design a codebook for User 1 in time slot 1.

Now we summarize the procedures to construct our codebook for User 1 in time slot 1:

1) For K bits of feedback, find $L_1 = 2^K$ two-by-one unit norm complex vectors which can maximize the minimum distance between any pair of vectors in the two-dimensional complex space. We denote all these vectors as $\psi_\mathbf{i}$, $i = 1, \ldots, L_1$.
2) Create a codebook $\boldsymbol{\Upsilon}_1$ that contains $L_1 = 2^K$ matrices satisfying $\boldsymbol{\Upsilon}_1[i] = \frac{1}{\sqrt{2}}[\psi_\mathbf{i}, \psi_\mathbf{i}]$.

It is easy to check that the created codebook satisfies all the conditions we need. Therefore, $|\mathbf{H}_{11}^1(1)|_F^2$ can be maximized if User 1 adopts the above codebook. Similarly, it can be shown that $\cos(\theta^1)|^2$ can also be maximized by using our proposed codebook. Once $|\mathbf{H}_{11}^1(1)|_F^2$ and $\cos(\theta^1)|^2$ are maximized at the same time, the coding gain will be maximized. Therefore, the above codebook is the optimal codebook that both User 1 and User 2 should adopt in each time slot.

VI. Simulation Results

In this section, we provide simulation results to evaluate the performance of the proposed scheme. First, we assume there are 2 transmitters each with 2 transmit antennas and 2 receivers each with 2 antennas. Each user uses our proposed scheme to transmit Alamouti codes to its receiver. Figure 3 presents simulation results using QPSK and 8 bits feedback. We can see that both users can achieve full diversity. In addition, because User 2 will cause interference to User 1. User 1 will have less coding gain compared with User 2. We compare the performance of our scheme with that of two other scenarios that can achieve interference cancellation. In the first scenario, we use TDMA and beamforming. That is, at each time slot, only one transmitter sends signals to one receiver using beamforming. 16-QAM is used to have the same bit-rate. In the second scenario, each user uses the multi-user detection(MUD) method [11] to send its codewords. The results show that our proposed scheme can achieve full diversity and symbol rate one. Note that we combine the array processing and space-time coding to avoid symbol rate loss. This does not mean that we cannot change the bit rate. We can always adapt the bit rate by changing the constellation

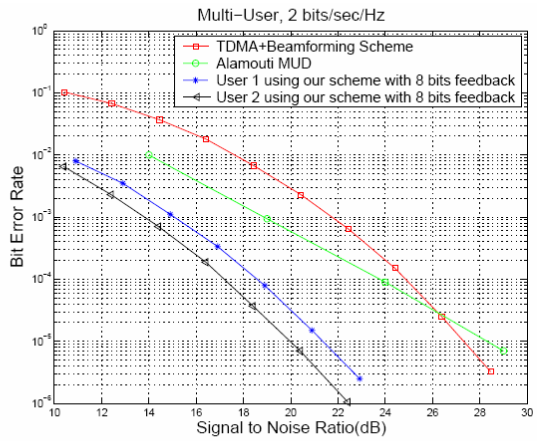

Fig. 3. Simulation results for 2 users each with 2 transmit antennas and 2 receivers each with 2 receive antennas. The constellation is QPSK and 8 bits feedback are used.

according to the channel condition. In comparison, the TDMA and beamforming method can achieve full diversity but the rate is one half. The MUD method can achieve full rate, but it cannot achieve full diversity. As shown in the figure, our scheme provides the best performance due to its high diversity and increased coding gain without any rate loss. Figure 4 shows the results when 8-PSK is used with 8 bits feedback. Similar results can be observed.

VII. Conclusions

In this paper, we propose a quantized feedback transmission scheme for Z channels by combining array processing and space-time coding to achieve full diversity and low decoding complexity. We first propose a precoding and decoding scheme to achieve interference cancellation. Then we propose a codebook and feedback design to achieve full diversity. To the best of our knowledge, this is the first scheme to achieve the highest possible diversity and low-complexity decoding simultaneously for Z channels when all users transmit simultaneously with only quantized feedback. We analytically prove that our scheme can achieve low-complexity decoding and full diversity. Simulation results validate our theoretical analysis.

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