

# Impact of Relay Location on the performance of Multi-Relay Cooperative Communication

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**Abstract—** Cooperative communications is a new wireless communication technique which allows single antenna mobiles to share their antennas and to produce virtual multiple antenna system. We investigate the impact of relay location on the SER and system capacity for multi relay (amplify and forward) cooperative network in terms of distance and obtained the best relay location analytically. The impact of relay location according to SER performance and system capacity is discussed by using linear topology with equal power allocation. From the simulation results, the better position of the relay is in the midway distance between the source and destination. The performance for the closest to source relay is good. On the other hand, the system performance degrades when the relay located close to destination. Through simulation results, it is shown that the error rate performance of the system increases quite remarkably with increase in the number of relays.

**Keywords-** Amplify and forward Relays, Maximal ratio combining, SER.

## I. INTRODUCTION

Wireless channel transmission suffers from random fluctuation in received signal at the destination known as fading. Diversity is one of the powerful techniques to mitigate fading. By using diversity technique, the transmitter sends more than one copy of the transmitted message so the receiver and it can use these multiple copies to detect the sent message correctly. It may be difficult to provide more than one antenna in wireless devices due to limitations like small terminal size and other factors. So a new way of realizing diversity has been introduced, which is known as cooperative diversity [1-5].

The author of the paper [6] analyzed the impact of relay location in terms of SER performance, instead of ergodic capacity or outage probability. However, the analytic network topology is single and has not given the theoretical basis of the optimal relay location. In paper [7], the main concern is that the theoretical analyses of SER in the single relay AF cooperative communication system. According to the SER formula in several equal-power allocated simple network topologies, the optimal relay location is trying to be gotten and the comparison of network topologies has done [8-10].

In this work, we have simulated a cooperative network consists of one source, 3 amplify and forward relays and one destination in both i.i.d (independent and identically distributed) flat Rayleigh fading channels for data with QPSK modulation by changing the location of the relays.

The simulation carried out in three cases (1 relay, 2 relays and 3 relays) and for each case, three positions of the relays which are close to source, close to destination and midway between source and destination are considered.

The paper organized as follows. Section II presents system model and description of amplify and forward relaying technique. Theoretical SER analysis over Rayleigh fading is presented in section III. The simulation results and analysis are explained in section IV and finally, section concludes the paper.

## II. SYSTEM MODEL

Consider a multi node cooperative diversity network as shown in Fig.1, which has a source S transmitting to a destination D through 3 cooperating relays. The source with the cooperation of relays transmits the information to destination through the amplify and forward (AF) protocol. All the links are mutually independent and subject to flat Rayleigh fading.

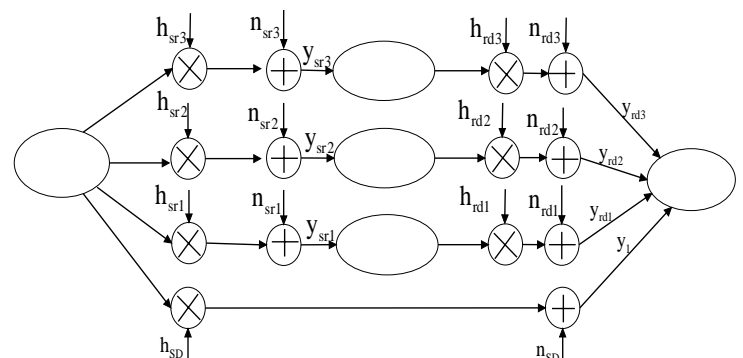


Fig.1. Three relay amplify and forward cooperative network model

There are two phases of operation.

### A. Broadcast phase :

The data is broadcasted by the source to the destination and to the N relays. The complex baseband received signal at the destination. The received complex base band signal at the destination and at the ith relay, are modeled, respectively, as

$$y_{s,d} = \sqrt{P_t} h_{s,d} x + n_{s,d} \quad (1)$$

$$y_{s,r_i} = \sqrt{P_t} h_{s,r_i} x + n_{s,r_i} \text{ for } i=1,2,\dots,N \quad (2)$$

$$h_{s,d} = l_{s,d} \cdot a_{s,d} \quad (3)$$

$$h_{s,r_i} = l_{s,r_i} \cdot a_{s,r_i}$$

Where  $P_t$  is the transmitted source power,

$h_{s,d}, h_{s,r_i}$  : are the channel coefficients of source node to destination node link and source node to  $i$ th relay node link, respectively.

$a_{s,d}, a_{s,r_i}$  : can be modeled as a zero mean, complex Gaussian random variables with variances  $\delta_{s,d}^2, \delta_{s,r_i}^2$  respectively. The path loss  $l_{s,d}$  is proportional to  $d_{s,d}^{-2}$ , where  $d_{s,d}$  is the distance from the source node to destination node. The path loss  $l_{s,r_i}$  is proportional to  $d_{s,r_i}^{-2}$ , where  $d_{s,r_i}$  is the distance between the source node and  $i$ th relay node.  $n_{s,d}, n_{s,r_i}$  are additive white Gaussian noise at the destination and  $i$ th relay respectively.

### B. Cooperation phase:

The  $N$  relays give cooperation to the source by transmitting the information to destination. Each relay acts as cooperative agent by helping the source (i.e. by amplifying the source signal and retransmits to the destination). The received data at the destination in cooperation phase due to the  $i$ th relay transmission is given by

$$y_{r_i,d} = \beta_i \sqrt{P_r} h_{r_i,d} x + n_{r_i,d} \text{ for } i=1,2,\dots,N \quad (4)$$

Where  $\beta_i$  is the amplification factor of relay given by

$$\beta_i \leq \sqrt{\frac{P_r}{P_t |h_{s,r_i}|^2 + N_0}} \quad (5)$$

Where  $h_{r_i,d}$  is the channel coefficients of  $i$ th relay to destination node link and is given by

$$h_{r_i,d} = l_{r_i,d} \cdot a_{r_i,d} \quad (6)$$

$a_{r_i,d}$  can be modeled as a zero mean, complex Gaussian random variables with variance  $\delta_{r_i,d}^2$ . The path loss  $l_{r_i,d}$  is proportional to  $d_{r_i,d}^{-2}$ , where  $d_{r_i,d}$  is the distance from the  $i$ th relay to destination node. The destination node implements a diversity combining technique to combine received signals.

Maximum Ratio Combining (MRC) is one of best the diversity combining strategies for the destination with knowledge of all channel fading parameters. The total signal at the destination in this case can be given by,

$$y_d = \sum_{i=1}^N h_{i,d}^* y_{id} \quad (7)$$

where,  $N$  is the number of branches at the combiner's input in the system,  $h_{id}$  is the conjugated channel gain corresponding to the received symbol  $y_{kd}$ . Accordingly, for our system model, number of inputs,  $N=3$ .

$$y_d = y_{sd} + \sum_{i=1}^N h_{i,d}^* y_{id} \quad (8)$$

In this analysis, the amplitude of the received signal from the channel (i.e., S to D, S to R and R to D) is modeled as a Rayleigh distributed. In other words, the channels in this model are assumed to be (i.i.d) independent and identically distributed Rayleigh fading channels.

### III. THEORETICAL ANALYSIS

If all of the channel links  $h_{s,d}, h_{s,r_i}, h_{r_i,d}$  are available, i.e.,

$\delta_{s,d}^2 \neq 0, \delta_{s,r_i}^2 \neq 0$  and  $\delta_{r_i,d}^2 \neq 0$  then when  $\frac{P_t}{N_0}$  and  $\frac{P_r}{N_0}$  go to infinity, the SER (symbol error rate) of the AF (amplify and forward) systems with M-PSK or M-QAM modulation can be tightly approximated as [4].

$$P_s \approx \frac{BN_o^2}{b^2} \cdot \frac{1}{P_t \delta_{s,d}^2} \prod_{i=1}^3 \left( \frac{1}{P_t \delta_{s,r_i}^2} + \frac{1}{P_r \delta_{i,d}^2} \right) \quad (9)$$

where, for M-PSK signal,  $b = \sin^2(\pi / M)$

$$B = \frac{3(M-1)}{8M} + \frac{\sin(\frac{2\pi}{M})}{4\pi} - \frac{\sin(\frac{4\pi}{M})}{32\pi} \quad (10)$$

and for QPSK modulation  $M=4, b=1$  &

$$B = \frac{9}{32} + \frac{1}{4\pi}$$

'No' is the power value of additive white Gaussian noise for each channel.  $P_t$  is the power of the signal transmitted from source;  $P_r$  is the power of the signal from relay.

When  $P_t = P_r = P/2$ ,  $P$  is the total transmission power  $P = P_t + P_r$ . Define

$$\gamma_1 = \frac{P_t}{N_o} \quad \gamma_2 = \frac{P_r}{N_o} \quad \gamma = \frac{P}{N_o}$$

$$\gamma_1 = \gamma_2 = \frac{\gamma}{2}$$

$$\overline{\gamma}_{s,d} = \frac{P_t |h_{s,d}|^2}{N_0} = \frac{P_t \delta_{s,d}^2 d_{s,d}^{-4}}{N_0} = \frac{1}{2} \gamma \delta_{s,d}^2 d_{s,d}^{-4} \quad (11)$$

$$\overline{\gamma}_{s,r_i} = \frac{P_t |h_{s,r_i}|^2}{N_0} = \frac{P_t \delta_{s,r_i}^2 d_{s,r_i}^{-4}}{N_0} = \frac{1}{2} \gamma \delta_{s,r_i}^2 d_{s,r_i}^{-4} \quad i=1,2,3 \quad (12)$$

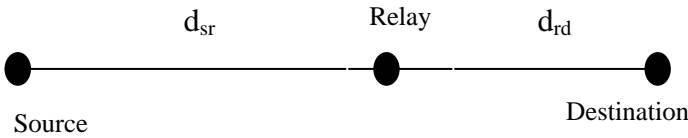
$$\overline{\gamma}_{r_i,d} = \frac{P_t |h_{r_i,d}|^2}{N_0} = \frac{P_t \delta_{r_i,d}^2 d_{r_i,d}^{-4}}{N_0} = \frac{1}{2} \gamma \delta_{r_i,d}^2 d_{r_i,d}^{-4} \quad i=1,2,3 \quad (13)$$

With channel model, (9) is transformed as follows through the use of (11), (12) and (13).

$$P_s \approx \frac{4B}{\gamma d_{s,d}^{-4}} \prod_{i=1}^3 \left( \frac{1}{\gamma d_{s,r_i}^{-4}} + \frac{1}{\gamma d_{r_i,d}^{-4}} \right) \quad (14)$$

#### A. Line Topology: Best location of relay

It is assumed that all the relay nodes are located at same position and the transmit power of each relay is same.  $d_{s,d}$  is the distance between source and the destination.  $d_{s,r_i}$ ,  $d_{r_i,d}$  are the distances between source to  $i$ th relay and  $i$ th relay to destination respectively.



Assuming  $d_{s,d}$  is a fixed value,  $d_{s,r_i} = d_{s,r} = p d_{s,d}$ . Let us say  $p \in (0,1)$  then  $d_{r_i,d} = d_{r,d} = (1-p)d_{s,d}$ .

The process of best location of relay is as follows

$$P_s \approx \frac{4B}{\gamma d_{s,d}^{-4}} \left( \frac{1}{\gamma (p d_{s,d})^{-4}} + \frac{1}{\gamma [(1-p)d_{s,d}]^{-4}} \right) \quad (15)$$

The second derivative of (99) with respect to 'd' is

$$\frac{4B}{\gamma^2 d_{s,d}^{-8}} [12p^2 + 12(1-p)^2] \quad (16)$$

(16)  $\geq 0$ , so the 'p' corresponding to the minimum value of (15) is exits. Now take the first derivate of (15) with respect to 'd' is and make equal to zero.

$$\frac{4B}{\gamma^2 d_{s,d}^{-8}} [4p^3 - 4(1-p)^3] = 0 \quad (17)$$

The root satisfying above equation for the constraint condition  $p \in (0,1)$  is **0.5**.

The distance between the source and destination is normalized to unity ( $R_{s,d}=1$ ). Along with analytical solution for best relay location, we represent a solution graphically by plot the SER (symbol error rate) of the system at different relay locations. We evaluate the symbol error rate affected by the relays at different locations. The cases considered for simulation are the relay nodes located near the source, near destination and midpoint between the source node and destination by increasing the number of relays.

$$d_{s,r_i} + d_{r_i,d} = d_{s,d} = 1 \quad (18)$$

#### B. Capacity analysis:

The ergodic capacity of the cooperative network with multi AF relay for equal power allocation is presented. It is assumed that the channel information is available at the receiver.

$$\overline{\gamma}_{s,d} = \frac{P_t |h_{s,d}|^2}{N_0} = \frac{P_t \delta_{s,d}^2 d_{s,d}^{-4}}{N_0} = \frac{1}{2} \gamma \delta_{s,d}^2 d_{s,d}^{-4} \quad (19)$$

$$\overline{\gamma}_{s,r_i} = \frac{P_t |h_{s,r_i}|^2}{N_0} = \frac{P_t \delta_{s,r_i}^2 d_{s,r_i}^{-4}}{N_0} = \frac{1}{2} \gamma \delta_{s,r_i}^2 d_{s,r_i}^{-4} \quad i=1,2,3 \quad (20)$$

$$\overline{\gamma}_{r_i,d} = \frac{P_t |h_{r_i,d}|^2}{N_0} = \frac{P_t \delta_{r_i,d}^2 d_{r_i,d}^{-4}}{N_0} = \frac{1}{2} \gamma \delta_{r_i,d}^2 d_{r_i,d}^{-4} \quad i=1,2,3 \quad (21)$$

$$\overline{\gamma}_{s,r_i,d} = \sum_{i=1}^M \frac{\overline{\gamma}_{s,r_i} \overline{\gamma}_{r_i,d}}{1 + \overline{\gamma}_{s,r_i} + \overline{\gamma}_{r_i,d}} \quad (22)$$

The ergodic capacity of the cooperative network with M relays is given by

$$C = \frac{1}{M+1} \log_2 \left( 1 + \bar{\gamma}_{s,d} + \bar{\gamma}_{s,r_i,d} \right) \quad (23)$$

$$C = \frac{1}{M+1} \log_2 \left( 1 + \frac{1}{2} \gamma \delta_{s,d}^2 d_{s,d}^{-4} + \sum_{i=1}^M \frac{2}{\gamma} \frac{\gamma \delta_{s,r_i}^2 \delta_{r_i,d}^2 d_{s,r_i}^{-4} d_{r_i,d}^{-4}}{d_{s,r_i}^{-4} \delta_{s,r_i}^2 + d_{r_i,d}^{-4} \delta_{r_i,d}^2} \right) \quad (24)$$

IV. SIMULATION RESULTS AND ANALYSIS

It is assumed that the users transmit their information through orthogonal channels with knowledge of CSI (channel state information) at the destination and perfect synchronization between the cooperating nodes in both of the phases (broadcast and cooperative). Thus to summarized various parameters are described in the following table.

TABLE.1.

Parameters	Specification
Number of bits	10 <sup>6</sup>
Bits per symbol	2
Number of source	1
Number of relays	3
Number of destination	1
Combining strategy	Maximal ratio combining
Channel	Flat Rayleigh
Modulation	Coherent QPSK

There are cases of simulation. In that, for every case(1 relay,2 relays and 3 relays), the location of relay ( $d_{s,d} - d_{s,r_i} - d_{r_i,d}$ ) has changed in 3 ways close source (1-0.2-0.8), close to destination (1-0.8-0.2) and midway between source and destination(1-0.5-0.5).

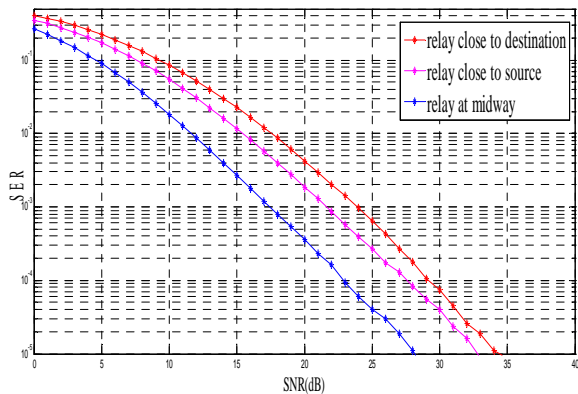


Fig.2. SER performance of amplify and forward cooperative scheme (1 relay case).

Figure 2 shows the effect of location of relay. As seen the best SER performance we get when the relay located midway between source and destination, or slightly closer to the

source. The further relay moves close to the destination then the performance is worse.

Similarly Fig.3 & Fig. 4 shows the BER performance when we increase the number of the relays. Finally the results are compared in Fig. 5. to see the effect of multiple relay and the distance on the system performance.

Finally the best preferable location of the relay is midway between the source and destination. The SER performance of this midway location of the relay for 3 cases (1 relay, 2 relays and 3 relays) has carried out.

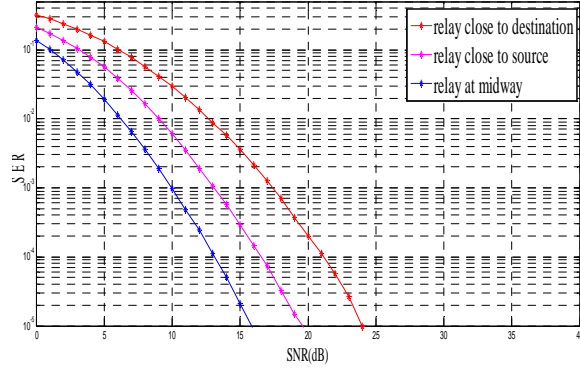


Fig 3: SER performance of amplify and forward cooperative scheme (2 relays case)

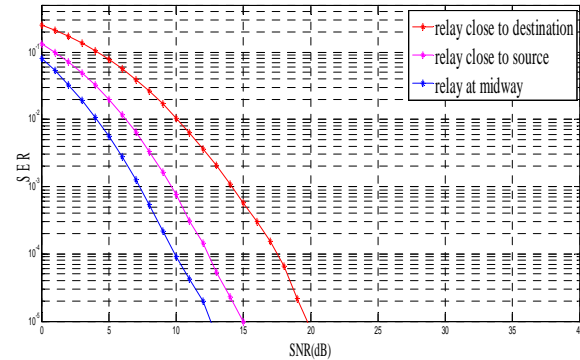


Fig 4: SER performance of amplify and forward cooperative scheme (3 relays case).

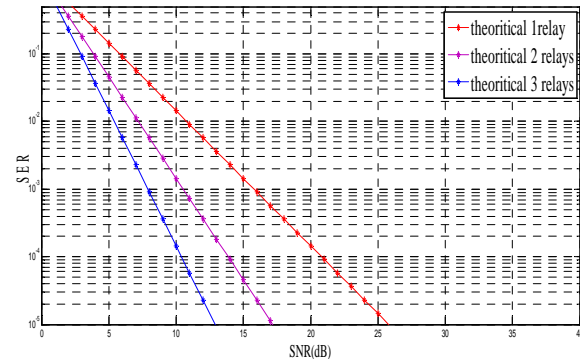


Fig 5: Theoretical SER performance of AF cooperative scheme (relay at midway position)

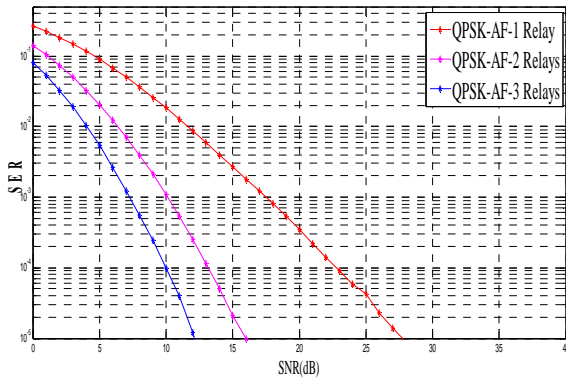


Fig. 6: Comparison of SER performance for multi AF cooperative scheme (relay located at midway).

The SER and ergodic capacity affected by the relay at different locations are presented. The simulation results in Fig.7 and Fig.8 show that there is increase in ergodic capacity and/or decrease in symbol error rate (SER) with increase in number of relays. Mainly we get maximum capacity and minimum SER at nearly midpoint between the source and destination.

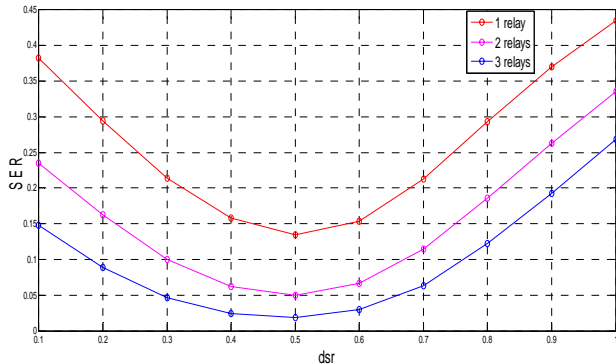


Fig.7: The SER depending on the relay locations with number of relays.

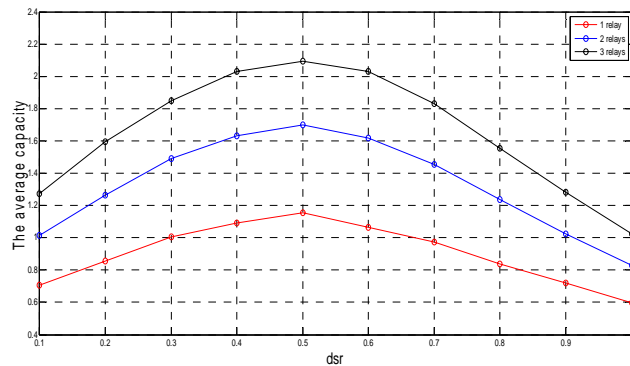


Fig.8: The average capacity depending on the relay locations with number of relays.

It is assumed that all relay nodes located at the same position, and the transmit power of each node is the same. Here the distance between source and destination is normalized to unity. Fig.7 depicts SER performance depending on different relay locations, i.e for different values of distance between source and the relay  $d_{sr} \in (0,1)$ . It is observed from the simulation that minimum SER, we get at middle  $d_{sr} = 0.5$ .

Due to the placing all relays at same position, we get minimum SER with e increasing number of relays.

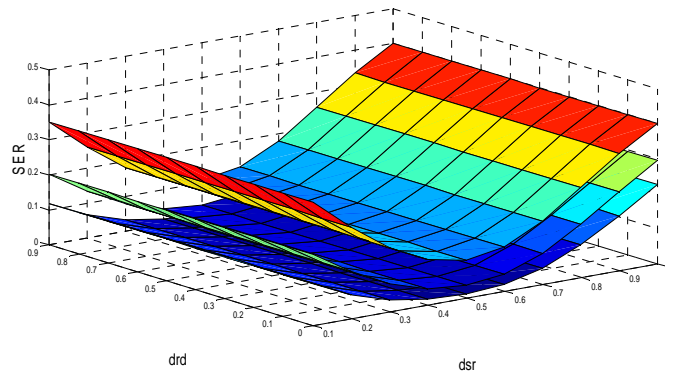


Fig.9.The SER depending on various relay locations  
 Fig.8 and Fig.9 depict 3d plots of SER and the average capacity depending on different relay locations respectively. In Fig.8, x-axis represents the distance from source to relay and y-axis represents the distance from relay to destination. It is observed that the SER of the system exhibits a symmetry property, and minimum SER value is achieved at the midpoint. Here each of the three cases, i.e, 1 relay, 2 relays and 3 relays, simulation for SER performance for different relay locations has been carried out at SNR of 5db.

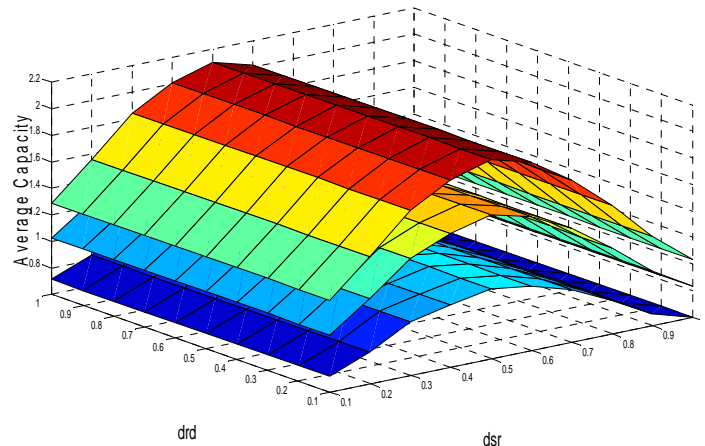


Fig.10.The average capacity depending on various relay locations

The cooperative networks in AF (amplify and forward) mode exhibits capacity symmetry with respect to the location of the relay.

## V. CONCLUSION

In this paper, we have obtained SER and ergodic capacity formulation in terms of relay location. Furthermore, the impact of relay location based on SER performance and capacity is discussed. The simulation results of the cooperative network, in which a source transmits the data with QPSK modulation to the destination using multiple amplify and forward relays with MRC combining strategy through flat Rayleigh fading channel, shows that the system performance increases with increase in number of relays. The best performance is achieved when the relay is positioned midway between the source and destination. An interesting future direction is to

simulate this system in nakagami and rician channels for MPSK signals.

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