SELECTION OF MIXED SAMPLING PLAN WITH CHAIN SAMPLING PLAN AS ATTRIBUTE PLAN INDEXED THROUGH MAPD AND IQL USING IRPD

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Abstract - This paper presents the procedure for the construction and selection of mixed sampling plan (MSP) using Intervened Random effect Poisson Distribution (IRPD) as a baseline distribution. Having the Chain Sampling Plan as attribute plan, the plans are constructed through indifference quality level (IQL) and maximum allowable percent defective (MAPD). Tables are constructed for easy selection of the plan

Key words and phrases: *indifference quality level, intervention, mixed sampling plan, maximum allowable percent defective, operating characteristic, poisson, intervened random effect poisson distribution.*

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I. INTRODUCTION

Mixed sampling plan is a two stage sampling procedure involving variables inspection in the first stage and attributes inspection in the second stage. If the variables inspection of the first sample does not lead to acceptance. Use of variables on the first sample with attributes on the second sample combines the economy of variables for quick acceptance on the first sample with broad nonparametric protection of attributes sampling when a questionable lot requires second sample.

Mixed sampling plans are of two types, which are independent and dependent plans. Independent mixed plans do not incorporate first sample results in the assessment of the second sample. Dependent mixed plans combine the results of the first and second samples in making a decision if a second sample is necessary.

Reference [13] proposed a method for determining the operating characteristics of mixed variables – attributes sampling plans, single sided specification and standard deviation known using the normal approximation. [2] ChSP-1 plan is an answer to the question whether anything can be done to improve the single sampling plans having c=0 without increasing the sample size. Clark (1960) has given a procedure for the selection of ChSp-1 plan given $P_{0\text{-}95}$ and $P_{0\text{-}10}$. Fred Frishman (1960) developed two types of chain sampling plans (ChSP-4 and ChSP-4A), which are extension of ChSP-1 plan in which rejection number greater than 1 is included. [15], [17], [18] has constructed tables for selection of ChSP-1 plan. Soundararajan and Govindaraju have made contributions in designing ChSP-1 plans. [19] have constructed Chain Sampling Plans (ChSP-1) indexed by inflection point and point of control. [1] has made contributions to mixed sampling plans with ChSP-1 plan as attribute plan. [4] constructed sampling plan of type CSP-T using MAPD and MAAOQ. The reduction in sample size is the principal advantage of these procedures over the traditional sampling procedures. [6], [7] derived an IRPD and constructed single sampling and conditional double sampling plans using MAPD and MAAOQ.

Reference [1] has studied the mixed sampling plans and reliability based sampling plans. [8] has constructed mixed variables – attributes sampling plans indexed through various parameters. [5] have constructed the mixed sampling plans using Poisson distribution as a baseline distribution. [12] have made contributions to mixed sampling plans for independent case. [9], [10], [11] also studied mixed sampling plan for independent case.

In the product control, the defective units are either rebuilt or replaced by new units during the sampling period. Quality engineers are always interested in improving the quality level of product to enhance the satisfaction of the customers and hence, they keep making changes in the production process. These actions trigger a change in the expected incidence of defective items in the remaining observational period. Any action for reducing the number of defectives during the sampling period is called an intervention and such intervention parameter ranges from 0 to 1.

In Intervened Random effect Poisson Distribution (IRPD), Poisson parameter is modified in two ways: one method is multiplying an intervention parameter ρ(a constant) and secondly, multiplying an unobserved random effect which follows Gamma probability distribution. The IRPD can be very useful to the quality and reliability engineers, who always make changes in the production system in the observational period of quality checking to ensure reliability of the system, because, the failure rate of the components may vary in different time intervals. The other areas of application of IRPD are queuing, demographic studies and process control and so on.

Reference [14] has used Intervened Poisson Distribution (IPD) in the place of Zero Truncated Poisson Distribution (ZTPD) for the study on cholera cases. [6], [7] introduced Intervened Random effect Poisson Distribution in the place of Poisson distribution for the construction of attribute sampling plans.

In this paper, using the operating procedure of mixed sampling plan (independent case) with chain sampling plan as attribute plan, tables are constructed using IRPD as a baseline distribution. The tables are constructed for mixed sampling plan (MSP) indexed through i) IQL ii) MAPD. The plan indexed through MAPD is compared with the plan indexed through IQL.

II. CONDITIONS FOR APPLICATIONS OF IRPD - MIXED SAMPLING PLAN

- Production process is modified during the sampling inspection by an intervention.
- Lots are submitted substantially in the order of their production.
- Inspection is by variable in the first stage and attribute in the second stage with quality defined as the fraction defective.
- Lot quality variation exists.

III. GLOSSARY OF SYMBOLS:

The symbols used in this paper are as follows:

p : submitted quality of lot or process

$P_a(p)$: probability of acceptance for given quality ‘ p ’

p_1 : submitted quality such that $P_a(p_0)$ = 0.50 (also called IQL)

p_* : maximum allowable percent defective (MAPD) n : sample size for each lot

n_1 : sample size for variable sampling plan

n_2 : sample size for attribute sampling plan

d : number of defectives in the sample

β_j : probability of acceptance for the lot quality ‘ p_j ’

β_j' : probability of acceptance under variables plan for percent defective ‘ p_j ’(with sample size n_1)

β_j'' : probability of acceptance under attributes plan for percent defective ‘ p_j ’(with sample size n_2)

z (j) : ‘z’ value for the j^{th} ordered observation

k : variable factor such that a lot is accepted if $\overline{X} \geq A = L + k\sigma$

IV. OPERATING PROCEDURE OF MIXED SAMPLING PLAN HAVING CHAIN SAMPLING PLAN AS ATTRIBUTE PLAN

Reference [13] has given the following procedure for the independent mixed sampling plan with lower specification limit (L) and standard deviation (σ).

- ❖ Determine the parameters of the mixed sampling plan n_1, n_2, k and i
- ❖ Take a random sample of size n_1 from the lot assumed to be large
- ❖ If a sample average $\overline{X} \geq A = L + k\sigma$, accept the lot
- ❖ If a sample average $\overline{X} < A = L + k\sigma$, take a second sample of size n_2 and count the number of defectives’d’ therein.
  - ➢ If the number of defectives d=0, accept the lot
  - ➢ If the number of defectives d≥2, reject the lot
  - ➢ Accept the lot if d=1 and if no defective units are found in the immediately preceding ‘i’ samples of size ‘n’

V. CONSTRUCTION OF MIXED SAMPLING PLAN HAVING CHAIN SAMPLING PLAN AS ATTRIBUTE PLAN USING IRPD.

Reference [13] has given the OC function of mixed sampling plan as

$$L(p) =$$

$$Pn_1 (\overline{X} \leq A) + Pn_1 (\overline{X} > A) \sum_{j=0}^{c} p\left(j;n_2 \right) \quad (1)$$

The above expression is given as

$$\beta_j = \beta_j' + (1- \beta_j') \beta_j'' \quad (2)$$

The operation of mixed sampling plans can be properly assessed by the OC curve for given values of the fraction defective. The development of mixed sampling plans and the subsequent discussions are limited only to the upper specification limit ‘U’. By symmetry, a parallel discussion can be made for lower specification limits.

The procedure for the construction of mixed variables – attributes sampling plans is provided by [13] for a given ‘n_1’, ‘k’ and a point ‘ p_j ’ on the OC curve is given below.

- Assume that the mixed sampling plans are independent
- Split the probability of acceptance (β_j) determining the probability of acceptance that will be assigned to the first stage. Let it be β_j'

- Decide the sample size n_1 (for variable sampling plan) to be used
- Calculate the acceptance limit for the variable sampling plan as

$$A = L + k\sigma = L + [z(p_j) + \{z(\beta_j')/\sqrt{n_1}\}]\sigma$$

, where L is the lower specification limit and

z (t) is the standard normal variate corresponding to 't' such that

$$t = \int_{z(t)}^{\infty} \left(\frac{1}{\sqrt{2\pi}}\right) e^{-u^2/2}\, du$$

- Determine the sample average $\bar{X}$. If a sample average $\bar{X} < A = L + k\sigma$, take a second stage sample size 'n_2' using attribute sampling plan.
- Split the probability of acceptance β_j as β_j' and β_j'' , such that $\beta_j = \beta_j' +$ (1- β_j') β_j'' and fix the value of β_j' .
- Now determine β_j'' , the probability of acceptance assigned to the attributes plan associated with the second stage sample as

$$\beta_j'' = (\beta_j - \beta_j')/(1 - \beta_j')$$

- Determine the appropriate second stage sample size 'n_2' from

$$P_a(p) = \beta_j'' \text{ for } p = p_j$$

Using the above procedure, tables can be constructed to facilitate easy selection of mixed sampling plan with chain sampling plan as attribute plan using IRPD as a baseline distribution indexed through MAPD and IQL.

Reference [6], [7] suggested the probability mass function of the IRPD as

$$P_a(p) = P(X=0) + [P(X=0)]^i P(X=1) \quad (3)$$

where

$$P(X=x) = \left[\frac{e^{-\theta}\theta^x}{(1+\rho\theta)^\alpha}\right] \sum_{l=0}^{x} \left(\frac{\rho}{1+\rho\theta}\right)^l \frac{(\alpha+l-1)!}{l!(x-l)!(\alpha-1)!}$$

, $\theta = \left(\frac{np}{1+\rho}\right)$, α=1

Using the above procedure, tables can be constructed to facilitate easy selection of MSP using IRPD as a baseline distribution. The tables furnished in this paper are for the case when α=1.

VI. CONSTRUCTION OF MIXED SAMPLING PLANS INDEXED THROUGH MAPD AND MAAOQ

MAPD, introduced by [3] and studied by [16] is the quality level corresponding to the inflection point of the OC curve. The degree of sharpness of inspection about this quality level 'p_*' is measured by 'p_t', the point at which the tangent to the OC curve at the inflection point cuts the proportion defective axis for designing, [16] proposed a selection procedure for single sampling plan indexed with MAPD and K= $\frac{p_t}{p_*}$.

Using the probability mass function of the IRPD, given in expression (3), the inflection point (p_*) is obtained by using $\frac{d^2P_a(p)}{dp^2} = 0$ and $\frac{d^3P_a(p)}{dp^3} \neq 0$. The n_2MAPD values are calculated for different values of i and ρ=0.8 for $\beta_*' = 0.24$ using c++ program and presented in Table 1.

The MAAOQ (Maximum Allowable Average Outgoing Quality) of a sampling plan is defined as the Average Outgoing Quality (AOQ) at the MAPD.

By definition AOQ $= p\ P_a(p)$ and MAAOQ = $p_*\ P_a(p_*)$

The values of MAPD and MAAOQ are calculated for different values of i and ρ for $\beta_*' = 0.24$ and the ratio $R = \frac{MAAOQ}{MAPD}$ is presented in Table 1.

Selection of the plan: For the given values of ρ, β_*', MAPD and MAAOQ, the ratio $R = \frac{MAAOQ}{MAPD}$ is found and the nearest value of R is located in Table 1. The corresponding i and n_2MAPD values are noted and the value of n_2 is obtained using $n_2 = \frac{n_2 MAPD}{MAPD}$.

Example 1: Given ρ=0.7, $\beta_*' = 0.24$, MAPD=0.0440 and MAAOQ=0.0170, the ratio $R = \frac{MAAOQ}{MAPD} = \frac{0.0170}{0.0440} = 0.3863$ is computed. In Table 1 the nearest R value is 0.3857, which is corresponding to i=9. The value of n_2MAPD=1.0225 is found and hence the value of n_2 is determined as $n_2 = \frac{n_2 MAPD}{MAPD} = \frac{1.0225}{0.0440} = 23$. Thus n_2=23, i=9 are the parameters of mixed sampling plan having ChSP as attribute plan using IRPD as a baseline distribution for the given values of ρ=0.7, MAPD=0.0440 and MAAOQ =0.0170.

VII. CONSTRUCTION OF MIXED SAMPLING PLANS INDEXED THROUGH IQL

The procedure given in section 5 is used for constructing the mixed sampling plan indexed through IQL (p_0). By assuming the probability of acceptance of the lot be β_0 =0.50 and β_0' =0.24, the $n_2 p_0$ values are calculated for different values of i and 'ρ' using c++ program and is presented in Table 2.

Selection of the plan: Table 2 is used to construct the plans when p_0, ρ, and i are given. For any given values of p_0, ρ, and i one can determine n_2 value using $n_2 = \frac{n_2 p_0}{p_0}$.

Example 2: Given ρ=0.7, p_0 =0.0318, i=4, β_0' =0.24. Using Table2, find $n_2 = \frac{n_2 p_0}{p_0} = \frac{0.1599}{0.0318} = 36$. For a fixed β_0' =0.24, the mixed sampling plan with ChSp as attribute plan is n_2=36, ρ=0.7, and i=9.

VIII. COMPARISON OF MIXED SAMPLING PLAN INDEXED THROUGH MAPD AND IQL

In this section MSP indexed through MAPD is compared with MSP indexed through IQL by fixing the parameters i and β_j'.

For the specified values of ρ, MAPD and MAAOQ with the assumption for $\beta_*' = 0.24$ one can find the values of i and indexed through MAPD. By fixing the values of i and find the value of p_0 by equating $P_a(p)$ = β_0 =0.50. For $\beta_0' = 0.24$, i and one can find the values of n_2 using $n_2 = \frac{n_2 p_0}{p_0}$ from Table 2. For different combinations of ρ, MAPD and

MAAOQ the values of i and n_2 (indexed through MAPD) and i and n_2 (indexed through IQL) are calculated and presented in Table 3.

Construction of OC curve

The OC curves for the plan ρ=0.7, n_2=23, i=9 (indexed through MAPD) and n_2=36, i=9 indexed through IQL) based on the different values of 'n_2 p' and $P_a(p)$ are presented in Figure 1.

Table 3: Comparison of the Plans

Given Values			Indexed Through MAPD		Indexed Through IQL	
MAPD	**MAAOQ**	**ρ**	**i**	**n_2**	**i**	**n_2**
0.0760	0.0260	0.8	4	16	4	24
0.0440*	0.0170	0.7	9	23	6	36
0.0230	0.0061	0.6	5	63	5	97

*OC curves are drawn

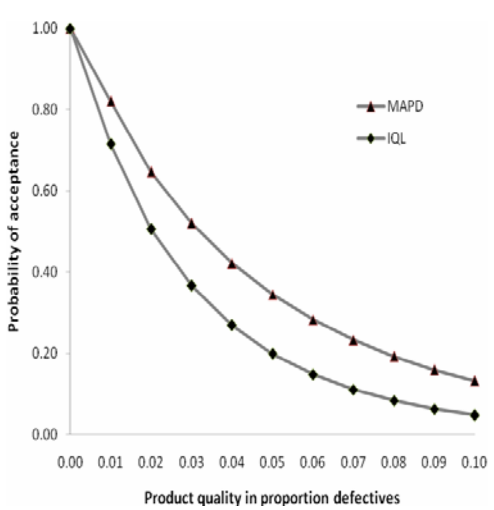

Fig1: OC curves for the plans (ρ=0.7, i=9, n_2=23) and (ρ=0.7, i=9, n_2=36)

IX. CONCLUSION

In this paper the construction of mixed sampling plan with Chain sampling plan as attribute plan indexed through the parameters MAPD and IQL are presented by taking IRPD as a baseline distribution. Further the plan indexed through MAPD is compared with the plan indexed through IQL. It is concluded from the study that the second stage sample size required for Chain sampling plan indexed through MAPD is less than that of second stage sample size of the Chain sampling plan indexed through IQL. If the floor engineers know the levels of MAPD or IQL, they can have their sampling plans on the floor itself by referring to the tables. This provides the flexibility to the floor engineers in deciding their sampling plans. Various plans can also be constructed to make the system user friendly by changing the first stage probabilities (β_*', β_0') and can also be compared for their efficiency.

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Table 1: n_2MAPD and n_2MAAOQ values for a specified values of i and different values of ρ for mixed sampling plan when $\beta_*' = 0.24$

ρ	i	β_*	β_*''	nMAPD	MAAOQ	$R = \frac{MAAOQ}{MAPD}$
0.8	**2**	0.5851	0.4540	0.9930	0.4508	0.4540
	3	0.5158	0.3628	1.1532	0.4183	0.3628
	4	0.4999	0.3419	1.1906	0.4070	0.3419
	5	0.5033	0.3464	1.1655	0.4037	0.3464
	6	0.5138	0.3602	1.1164	0.4021	0.3602
0.7	**2**	0.5466	0.4034	1.1046	0.4455	0.4034
	3	0.4817	0.3180	1.2812	0.4074	0.3180
	4	0.4677	0.2996	1.3241	0.3967	0.2996
	5	0.4823	0.3188	1.2455	0.3970	0.3188
	6	0.4948	0.3352	1.1850	0.3972	0.3352
	7	0.5080	0.3526	1.1257	0.3969	0.3526
	8	0.5210	0.3697	1.0710	0.3959	0.3697
	9	0.5332	0.3857	1.0225*	0.3943	0.3857
0.6	**2**	0.5078	0.3523	1.2288	0.4329	0.3523
	3	0.4476	0.2731	1.4278	0.3899	0.2731
	4	0.4355	0.2572	1.4764	0.3797	0.2572
	5	0.4401	0.2632	1.4453	0.3804	0.2632
	6	0.4507	0.2772	1.3846	0.3838	0.2772
	7	0.4630	0.2934	1.3192	0.3870	0.2934
	8	0.4759	0.3103	1.2552	0.3901	0.3103
	9	0.4880	0.3263	1.1982	0.3909	0.3263
	10	0.5001	0.3422	1.1444	0.3916	0.3422

Table 2: n_2 IQL values for a specified values of i and ρ of mixed sampling plan when β_0 =0.50 and β_0' =0.24

	ρ Values								
i	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
2	1.1835	1.1948	1.2092	1.2252	1.2419	1.2587	1.2755	1.292	1.3081
3	1.1161	1.1267	1.1402	1.1551	1.1707	1.1864	1.2021	1.2175	1.2325
4	1.0912	1.1015	1.1146	1.1292	1.1443	1.1597	1.1750	1.1900	1.2047
5	1.0821	1.0923	1.1053	1.1197	1.1348	1.1500	1.1651	1.1800	1.1946
6	1.0788	1.0890	1.1020	1.1164	1.1314	1.1465	1.1616	1.1765	1.1910
7	1.0777	1.0879	1.1009	1.1152	1.1302	1.1454	1.1604	1.1753	1.1898
8	1.0773	1.0875	1.1005	1.1148	1.1298	1.1449	1.1600	1.1748	1.1893
9	1.0772	1.0874	1.1003	1.1147	1.1297	1.1448	1.1599*	1.1747	1.1892
10	1.0771	1.0873	1.1003	1.1146	1.1296	1.1448	1.1598	1.1746	1.1891

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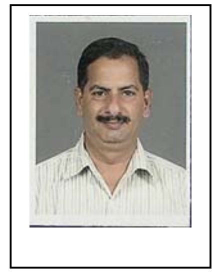

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