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Frequency Offset Estimation in OFDM Systems using Threshold based Scheme

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Abstract— **The Performance of the Orthogonal Frequency Division Multiplexing (OFDM) systems degrade due to Doppler frequency or frequency drift between transmitter and receiver oscillator which causes frequency offset and hence leads to a number of impairments in the received signal, thereby making its detection incorrect. In this paper, a threshold-based frequency offset estimation algorithm for orthogonal frequency division multiplexing (OFDM) systems is further analyzed. The algorithm shows robust performance in the presence of timing offset. The threshold used in correlation and coherence phase bandwidth lowers the computation complexity when compared with the existing schemes. Through a detailed analysis and simulation over AWGN channel, it has been shown that the scheme analyzed maintains the same level of estimation performance with lower complexity than the existing schemes.**

Keywords- *Frequency offset, OFDM, synchronization, coherence phase bandwidth (CPB)*

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has been chosen as the most promising transmission technique in several mobile radio communication systems, such as the European Digital Audio Broadcasting (DAB) system and the Digital Video Broadcasting (DVB) system [1]. One of the drawbacks of OFDM has proven to be its high sensitivity to frequency offset. This frequency offset is usually divided into an integer part, which is a multiple of the subcarrier spacing, and a fractional part, which is less than one half of the subcarrier spacing. The former results in a shift of the subcarrier indices, while the latter causes a number of impairments, including attenuation and rotation of each of the subcarriers and inter-carrier interference (ICI) between subcarriers [4].

In the literature several schemes have been proposed in [2]-[6] to estimate the frequency offset. The frequency offset estimation schemes can be classified into two categories: fractional frequency offset estimation schemes [4], [5], [6] and integer frequency offset estimation schemes [2], [3]. For integer frequency offset estimation, an estimation scheme was proposed in [2] (Nogami's scheme) using the cross correlation between the received and locally generated training symbols. However, the scheme in [2] is very sensitive to the timing offset. Thus, in [3] (Bang's scheme), an estimation scheme robust to the timing offset was proposed considering CPB in its estimation process. However, the scheme in [3] still has the problem that the complexity in implementation rapidly increases, as the frequency offset estimation range increases.

The frequency offset estimation algorithm analyzed in this paper is robust to a symbol timing offset and has lower computational complexity when compared with Nogami's [2] and Bang's [3] schemes.

II. SYSTEM MODEL

In an OFDM system, the complex baseband OFDM signal after the IFFT block at the transmitter can be expressed as

$$\mathbf{x(n)} = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi nk}{N}} \quad , n = 0, 1, 2 \ldots N-1$$

where N is the total number of subcarriers, X(k) denotes the transmitted quadrature amplitude modulation (QAM) or M-ary phase-shift keying (PSK) modulated symbol on the subcarrier k with k = 0, 1, 2, …, N −1.

In the presence of frequency and timing offsets, the received OFDM signal $\mathbf{r(n)}$ can be expressed as

$$\mathbf{r(n)} = \mathbf{x}(n - n_o)e^{j\frac{2\pi[(n-n]_o)f_o}{N}} + \mathbf{w}(n) \qquad (2)$$

where f_o and n_o represent the frequency and timing offsets normalized to the subcarrier spacing and sample interval 1/Ts, respectively, Ts is the symbol duration and $\mathbf{w(n)}$ is the zero-mean complex additive white Gaussian noise (AWGN).

The frequency offset f_o can be divided into an integer part and a fractional part as

$$f_o = \epsilon + \Delta f_f$$

where ϵ is the integer part of f_o and f_f $\in$ [−0.5, 0.5) is the fractional part of f_o.

We estimate integer frequency offset and assume that the fractional part f_f is known and perfectly compensated. The received symbol is demodulated using FFT operation. The *k*th FFT output *R(k)* can be expressed for $k = 0, 1, 2, \ldots, N-1$ as

$$R(k) = X(k-\epsilon)e^{-j\frac{2\pi[(k-\epsilon)]n_0}{N}} + W(k)$$

where W(k) is the FFT of w(n).

III. NOGAMI'S SCHEME

To estimate the integer frequency offset, Nogami's scheme examines the correlation value between the known training symbol and the cyclically shifted version of the received training symbol, and then, obtain the integer frequency offset estimate $\hat{\epsilon}$ as

$$\hat{\epsilon} = \arg\max_{d}\left\{\sum_{k=0}^{N-1} Z^{*}(k)R(k+d)_N\right\} \quad (4)$$

where d is the amount of cyclic shift, (.)N is the modulo-N operator, R(k) is the received pilot symbol and Z(k) is the known pilot symbol. Nogami's algorithm was proposed on the assumption of perfect timing synchronization, so it is not appropriate for cases where a symbol timing offset exists. Thus, its estimation performance degrades in the presence of timing offset.

IV. COHERENCE PHASE BANDWIDTH

The coherence phase bandwidth BW_c is defined as the maximum integration range according to the allowed symbol timing offset T_{allow} , which can be guaranteed in the coarse timing synchronization. The coherence phase bandwidth is expressed as follows:

$$BW_c \approx \frac{1}{2\,T_{allow}} BW \quad (5)$$

where BW is N . In fact, BW_c represents the integration range within which the correlation value reaches the first peak of the plot for $\tau = T_{allow}$ in Fig. 1.

Fig. 1 shows the plots for the correlation values from (4) with respect to the integration range for several values of the symbol timing offset. For a symbol timing offset within half of a sample period, the correlation value increases monotonically depending on the increment of the integration range. However, for a symbol timing offset of more than one sample period, the correlation value oscillates according to the increment of the integration range and is 0 or near 0 for a certain integration range.

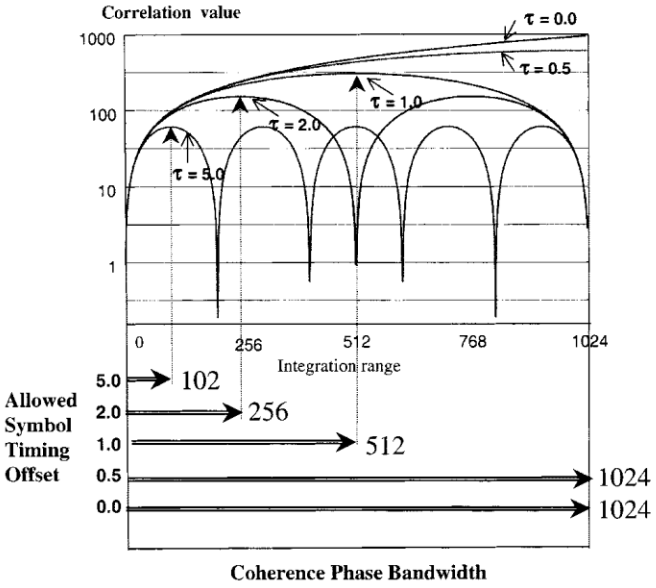

Fig. 1. Correlation values versus the integration range for several values of symbol timing offset. The bottom lines represent the coherence phase bandwidth according to τ . The number of subcarriers is 1024.

In particular, when the integration range is N, the correlation value is 0 or near 0 if the symbol timing offset is in the proximity of an integer value. This means that Nogami's algorithm may fail to estimate the coarse frequency offset if a certain symbol timing offset exists. However, one can also find that for any symbol timing offset larger than 1, there exists a certain integration range (smaller than N) for which the correlation values increases monotonically.

If we let the integration range be BW_c , no other correlation value with the integration range of BW_c falls to zero if $\tau < T_{allow}$. This concept was utilized in Bang's scheme.

V. BANG'S SCHEME

Bang's scheme adopts the concept of the coherence phase bandwidth for the purpose of weakening the effect of symbol timing offset. The coherence phase bandwidth is obtained by (5). The total system bandwidth is divided into a number of blocks with coherence phase width i.e.

$$K = \frac{BW}{BW_c} \quad (6)$$

where K is the number of divided blocks.

In this algorithm, the correlation is calculated individually within each small block and then summed. The estimated coarse frequency offset is obtained as

$$\hat{\epsilon} =$$

$$\arg\max_{d} \left\{ \sum_{m=0}^{N-1} \left| \sum_{k=0}^{BW_c-1} Z^*(k+mBW_c)R(k+mBW_c+d)_N \right| \right\} \quad (7)$$

VI. THRESHOLD BASED FREQUENCY OFFSET ESTIMATION

When we calculate one CPB block in (7), correlation value C can be expressed as

$$C = \left| \sum_{k=0}^{CPB-1} Z^*(k)R(k+d)_N \right| \quad (8)$$

Where $CPB = \frac{1}{2n_o^t}N$ and $\mathbf{n_o}$ is the maximum tolerable timing offset value.

Assuming $d = \epsilon$ and ignoring AWGN components in (8), the correlation value C can be expressed as

$$C = |z_k|^2 \left| \sum_{k=0}^{CPB-1} e^{-\frac{j2\pi \mathbf{n_o} k}{N}} \right| \quad (9)$$

The value C in (9) is minimum when $\mathbf{n_o} = n_o^t$, Therefore, the minimum correlation value $C_{\min}$ can be expressed as

$$C\min = |z_k|^2 \left| 1 - j\cot\left(\frac{\pi n_o^t}{N}\right) \right| \quad (10)$$

We also express the minimum value of full correlation in a similar manner as

$$C\min = 2n_o^t |z_k|^2 \left| 1 - j\cot\left(\frac{\pi n_o^t}{N}\right) \right| \quad (11)$$

In this paper, we use a threshold η as a half of $C_{full\min}$

$$\eta = n_o^t |z_k|^2 \left| 1 - j\cot\left(\frac{\pi n_o^t}{N}\right) \right| \quad (12)$$

The above algorithm works as follows:

1. Calculate CPB.
2. Then calculate η from equation (12).
3. The correlation value is acquired using the CPB from (8).
4. If the correlation value exceeds η, then d is decided to be the correct estimate of ϵ. Otherwise, the received signal is cyclically shifted by d and the above procedure is repeated.

If the expected frequency offset range is sufficiently large, the proposed scheme will generally have about a half computational complexity when compared with others. Also, it should be noted that the proposed scheme does not require additional memory for correlation values unlike Nogami's and Bang's schemes.

VII. SIMULATION RESULTS

The simulation has been performed over MATLAB. Fig. 2 shows the correlation values of Nogami's scheme when $d = \epsilon$ and $N = 1024$ in the absence ($\tau = 0$) and presence $\tau = 0.5$, 1, and 2 of timing offset. In the absence of timing offset, the correlation value increases as the integration range increases, on the other hand, in the presence of timing offset, the correlation value oscillates.

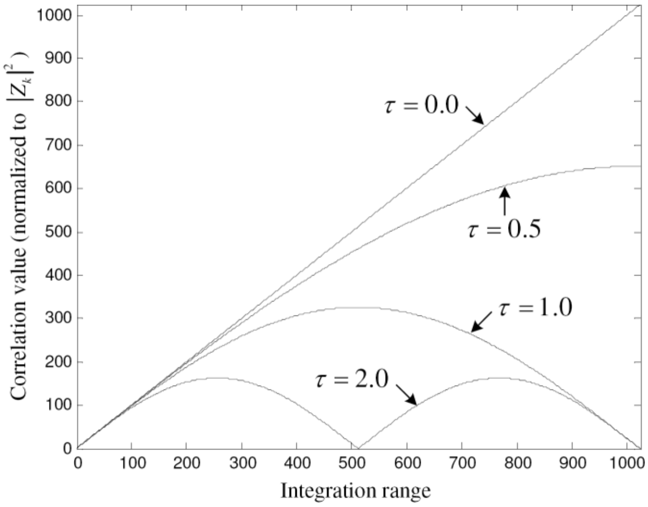

Fig. 2. The correlation values of Nogami's scheme in the absence and presence of timing offset when $d = \epsilon$ and $N = 1024$.

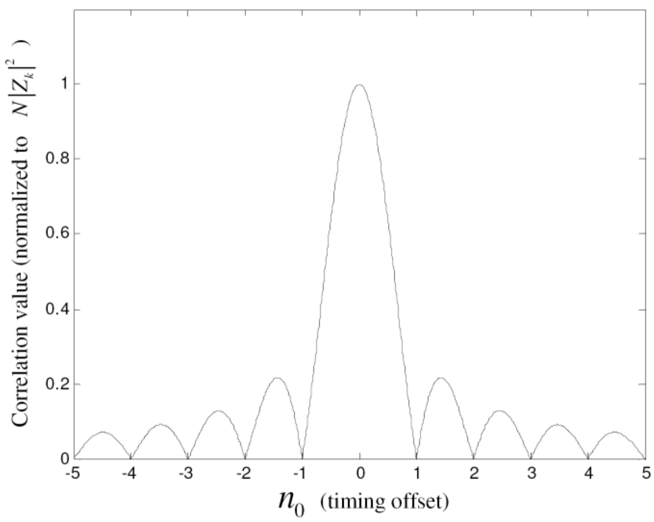

Fig. 3. The correlation value of Nogami's scheme as a function of timing offset $\mathbf{n_o}$ when $d = \epsilon$ and N=1024

Fig. 3 shows Nogami's correlation function as a function of timing offset. From the figure, we can see that the correlation value becomes smaller as the value of timing offset increases.

Fig. 4, when the integration range is BW_c the correlation value of Bang's scheme increases monotonically as the value of integration range increases. Fig 5. shows Bang's correlation function as a function of timing offset. From the figure, we can observe that the correlation function has no zero-crossing point unlike Nogami's correlation function. Thus, Bang's scheme would have the better estimation performance than Nogami's scheme in the presence of timing offset. However, Bang's scheme has high computational complexity since the correlation values are calculated for all possible values of *d*.

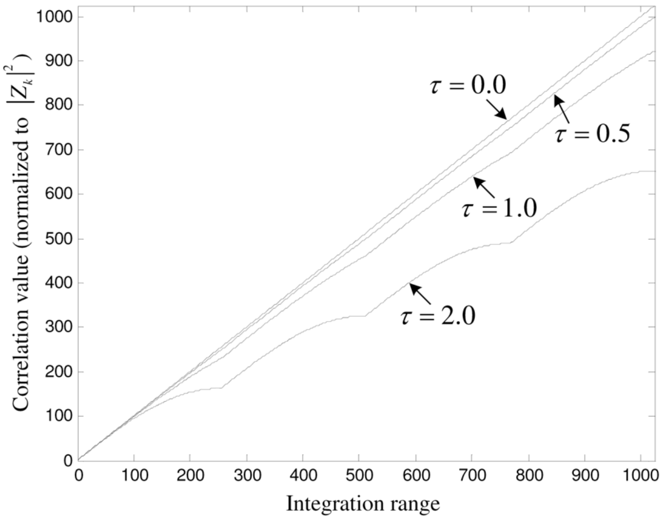

Fig. 3. The correlation values from Bang's scheme versus the integration range for several values of timing offset (when $d = \epsilon$, $N = 1024$, and CPB= 256).

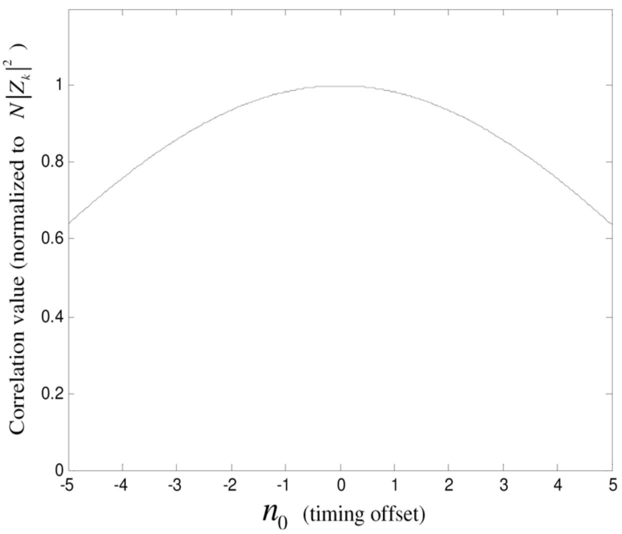

Fig. 4. The correlation value from Bang's scheme plotted as a function of timing offset *n*0 (when $d = \epsilon$ and CPB = $N/10$).

In this paper, we use AWGN as the channel model for performance comparison. The signal-to-noise ratio (SNR) of AWGN channel model is 5 dB. The frequency offset used in this simulation is an integer value in [0, 500], $N = 1024$, and CPB= $N/32$.

Table 1, for $L \gg 1$, where L is the frequency offset range, illustrates that the proposed scheme exhibits approximately half computational complexity compared with the Nogami's and Bang's schemes.

Table 2 depicts that the proposed scheme exhibits significantly improved frequency offset estimation performance over Nogami's scheme and almost comparable performance to Bang's scheme in the presence of the timing offset.

TABLE I
COMPLEXITY COMPARISON FOR THE SCHEMES

Scheme	No. of complex multiplication	No. of comparison operation	Size of memory for correlation value
Nogami's Scheme	LN	L-1	L
Bang's Scheme	LN	L-1	L
Threshold based scheme	$\frac{L+1}{2}N$	$\frac{L+1}{2}$	-

TABLE II
ACCURACY COMPARISON OF THE FREQUENCY OFFSET ESTIMATION SCHEMES IN THE AWGN CHANNEL MODEL.

Timing Offset(samples)	Nogami's Scheme	Bang's Scheme	Threshold based Scheme
0	100	100	100
1	0	100	100
2	0	100	100
5	0	100	100

VIII. CONCLUSION

In this paper, we have analyzed frequency offset estimation scheme based on threshold and CPB. The simulation results show that the scheme exhibits much lower complexity than the conventional schemes by using threshold for decision making. It maintains the same level of estimation performance when compared with the existing schemes. At the same time it is robust against symbol timing offset by using the concept of CPB.

IX. REFERENCES

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