Dynamic Interval Graph Coloring for Channel Assignment in Wireless Cellular Networks

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Abstract—**Wireless Cellular Networks are most often modeled as a graphs and Channel Assignment Problems (CAP) can be formulated as graph coloring problms. Assignment of channels to the calls in the networks is a discrete-time event process involving the correct mapping of the channels to the caller-receiver pairs. Efficient usage of limited channels adopt the principles of reusability of same channels for some calls at same time instance. In this paper, an efficient coloring technique is proposed and we apply it to the identified interval-graph dynamically.**

Keywords- graph coloring;, interval graph; cellular network; channel assignment.

I. INTRODUCTION

Wireless cellular networks are wireless telephone networks that has a set of m cells in a geographical region denoted as C_i for i = 1, 2.... m [1]. Each cell has a Base Station (BS) which controls all the incoming and outgoing calls within its range. The cells covering the entire networks have headquarter known as Mobile Switching Center (MSC). MSC maintains a database for all the mobile users within the networks. The calls established in the networks consist of a pair of communicating nodes: a caller in one end and a receiver in the other end. This is possible only when both caller and receiver are allocated with channels.

Considering the cellular networks correspond to an undirected graph, the set of vertices can be assumed to be callers or receivers and the set of edges are assumed to be channels among the callers and receivers. A graph coloring problem which is NP-Complete [2] may be used to solve the problem of channel assignment in the networks. Each of the caller-receiver pairs of the networks may be interpreted as an interval in the real line. So the graph is known as interval graph. Interval coloring is an alternative way of solving the assignment problem in the networks.

In this paper, we referred an existing method, for the identification of intervals in a given real-time graph. Then applying our proposed coloring algorithm to these intervals we claim that channels assigned to the calls are perfect.

Rest of this paper is organized as follows: section II presents problem formulation. Methodology used in our strategy and the proposed algorithm is described in section III. Section IV concluded the paper with future scope.

II. PROBLEM FORMULATION

Given a wireless cellular network with a fixed number of channels, the problem is to assign channels to all the calls such that adjacent, co-channel and cosite channel interference must be respected. At a discrete time interval t, a set of s telephone calls are received. All the calls are assumed to be originated within the network and the receivers are also assumed to be within the same network. Each call requires two channels, one for the caller and another for the receiver. Therefore, the task of assigning the channels on real-time basis to the pairs of caller-receiver rests on MSC. The channels are assigned to the mobile users on the first-come-first-served basis. If none of the channels are available at a given time slot then the call is placed in a queue and the first pair of available channels are immediately assigned once they become available. The question in this study is in what way can the channels be allocated to meet this requirement?

A graph G is called an interval graph if there exists one to one correspondence among the vertices and set of intervals on the real line such that two vertices are adjacent in G if and only if the corresponding intervals have a nonempty intersection. The set of intervals assigned to the vertices of G is called a realization of G. If the set of intervals can be chosen to be inclusion-free, then G is called a proper interval graph. Proper interval graphs have been studied extensively in the literatures [3], [4], and several linear time algorithms are known for their identification and realization [5] [6]. This paper deals with the problem of identification and representing dynamically changing proper interval graphs. Later we apply coloring technique to this interval graph.

III. METHODOLOGY USED

Our model proposes the task of assigning channels to the pairs of callers and receivers by the MSC. A call between the caller and the receiver is possible only when each of them obtains a channel from the MSC. The initial task is to identify the given network is an interval graph. We referred the algorithm given in [7] to identify an interval graph. They used the concept of PQ-tree to derive an interval graph.

The channel assignment strategy in this paper follows from the graph transformation model. This strategy is outlined as follows:

- Initially, a cellular network in which requests for channels among communicating mobile users is known as base graph *G*.
- *G* is converted to interval graph *S* using PQ-tree technique as given in [7].
- The endpoints of each interval in *S*, represents the channels for the caller-receiver pairs.
- Finally, an efficient coloring technique (proposed algorithm) is applied to *S* to color the graph without violating the perfect coloring constraints.
- In this way the colors also called channels are being assigned to the users for allowing communication among them.

A. Identification of Interval Graph

In the context of channel assignments, the same strategy outlined above is applied. We illustrate the channel assignment model through the transformation from *G* to S using Figure 1 as an illustration.

The figure shows a cellular network of 19 cells arranged in a circular formation. With the center at Cell 1 and having 2 units of cells as its radius, this circular arrangement is referred as H_2. At an instance of time *t* , a request for a total of six calls are received by MSC, and the calls are shown as the caller-receiver pairs in the form of straight lines in the figure. Each line in the figure shows the caller-receiver position with the cell number (end point) as the origin or destination of the calls.

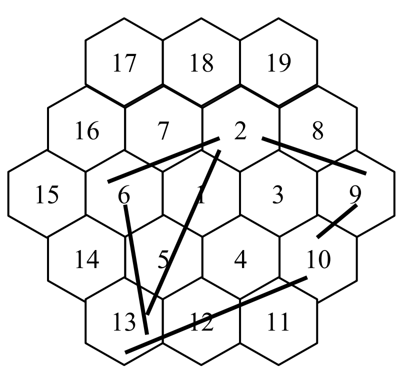

Figure. 1 Base Network (cellular network showing active calls from pairs of mobile users)

Figure 2 shows a connected graph *G* derived from the cellular diagram in Figure 1. The graph is formed with the host cell number as the node number.

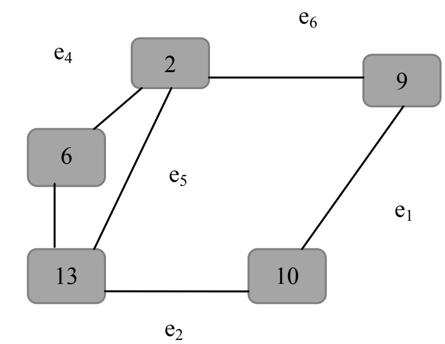

Figure 2 Graph *G* formed from Figure 1.

Algorithm produces N points or N / 2 intervals in S from *G,* as shown in Figure 2. The dark boxes in the figure are the zones whose zone number corresponds to the node number in *G.* Each zone has between one to four points, which correspond to the degree of the node in *G.*

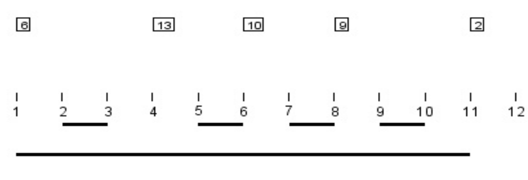

Fig-2 Initial order zones and intervals in S

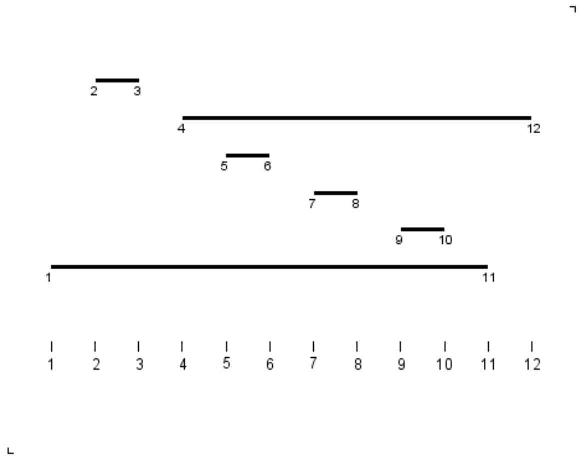

Fig-3 Final Order zones and intervals in S

B. Coloring Interval Graph

A vertex coloring of a graph G = (V, E) is a map c: V→ S such that c (u) ≠ c (v) whenever u and v are adjacent. The elements of the set S are called the available colors. All that interest us about S is its size: typically, we shall be asking for the smallest integer k such that G has a k coloring, a vertex coloring c: V→ {1,2… k}. This k is the (vertex-) chromatic number of G, denoted by χ(G). A graph G with χ(G)≤ k is called k-colorable. How we determine the chromatic number of a given graph? How can we find a vertex coloring with as few colors as possible? One obvious way to color a graph G with not too many colors is the following greedy algorithm: starting from a fixed vertex enumeration v_1, v_2,… v_n of G, we consider the vertices in turn and color each vi with the first available color - e.g., with the smallest positive integer not already used to color any neighbor of vi among v_1, v_2… v_{i-1}. As we will prove in the following theorem, in this way, we never use more than Δ(G) + 1 colors (Δ (G) = max {d (v) | v ∈ V} is the maximum degree of G.), even for unfavorable choices of the enumeration v_1, v_2… v_n. Notice that if G is complete or an odd cycle, then this is even best possible.

C. Interval Graph Coloring

Let S is the interval graph identified from the given graph G. Each edge $e_i \in$ S, known as an interval on the real line whose end vertices map with unique pair numbers on the real line. Let C is a finite set of colors to be assigned to each pair of vertices of an edge such that no vertex belong to same or adjacent cell should have same color and if *v* is adjacent to vertex *u* then *col(v) ≠ col(u),* where *col(u)* is known as color of vertex *u*.

Algorithm:
Input: Interval graph S.

1. **While** | C| ≠ ϕ // C is set of colors
2. **for** each i = 1 to N / 2
3. **for** each edge $e_i \in$ S
4. select vertex *u* without assigned by any color then *u* ⟵ color *c* such that *col(v) ≠ c* where vertex *v* is adjacent to *u*.
5. **end for**
6. **end for**
7. **end while**

Theorem: Any interval graph G can be optimally colored using the greedy strategy

.

Proof.
Consider the following procedures for the interval graph coloring.

- Start the intervals in increasing order of their starting times.
- Consider an interval *I* with given color *c*. If color *c* has been assigned to an earlier interval, and *I* can be colored by *c* without violating constraint on coloring, then color the interval *I* with color *c*.
- Otherwise, color the interval *I* with a new color.

The above procedure claims a proper coloring. Since this greedy strategy uses *k* colors to color a given set of intervals *I*, it must be the case that there are *k* intervals that mutually intersecting (otherwise, no new color would had been used). Therefore, *k* is the minimum number of colors that could be used for color *I*.
The running time of the above set of procedure is linear for processing all intervals, in addition to the sorting time (or *O* (*n* log *n*)).
Corollary If G is an interval graph, then its chromatic number is equal to its clique number
(The size of the largest complete sub-graph in G) namely, χ(G) = ω(G).

IV. CONCLUSION AND FUTURE SCOPE

As the graph changes its topology dynamically, this paper proposes a dynamic interval graph coloring for assigning channels in wireless cellular networks. The concept is based on the interval graph model of the network. The connected graph is made up of active calls whose nodes represent the cells in the cellular network. Each edge in the graph is an active call whose end points are the pair of caller-receiver in the call. This active call is mapped as an interval in the network. Our future study will focus on the interference of channels (both adjacent and co-channel) in this technique.

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