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Teleportation of Qubits - A Simulation Study

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***Abstract*—Theoretical analysis of quantum cryptography requires the role of teleportation. At most, all of the security proofs work by reducing the considered quantum cryptography protocol to an equivalent entanglement based protocol which uses quantum teleportation. In this paper, a well-established concept, the quantum teleportation is simulated using an open source package qlib.**

Keywords-quantum cryptography; teleportation; qubit;

I. INTRODUCTION

Quantum teleportation is a process by which quantum information (the exact state of photon) can be transmitted from one location to another, with the help of classical communication and previously shared quantum entanglement between the sending and receiving location. Actually quantum teleportation is not a form of transportation, but of communication. It concerns only the transfer of information, i.e., it provides a way of transporting a qubit from one location to another, without having to move a physical particle along with it [1].

A. Qubit

A single qubit at an arbitrary time'*t*' can be expressed as $|\Psi(t)\rangle = \alpha(t)|0\rangle_{+}\beta(t)|1\rangle$ and it can be manipulated by acting a physical operator on it. To evolve the state the Schrodinger time dependent equation can be used [2].

$$i\hbar \frac{\partial (\Psi(t))}{\partial t} = H\Psi(t)$$

H is a quantum observable quantity and is called as Hamiltonian.

$$H = \Delta\sigma_x + \chi\sigma_{y+}\Gamma\sigma_z = [■(■((\&\Delta - i" (" @\Delta + i" (" \& - "(")\&)] \quad (1)$$

Δ,χ,Γ are some parameters for the real physical environment and σ_x, σ_y,σ_zPauli's matrices.

II. QUANTUM TELEPORTATION

The prerequisites for quantum teleportation are a qubit that is to be teleported, a conventional communication channel capable of transmitting two classical bits (i.e., one of four states), and means of generating an entangled EPRpair of qubits**,** transporting each of these to two different locations, say Alice (A) and Bob (B) , performing a Bell measurement on one of the EPR pair qubits, and manipulating the quantum state of the other of the pair.

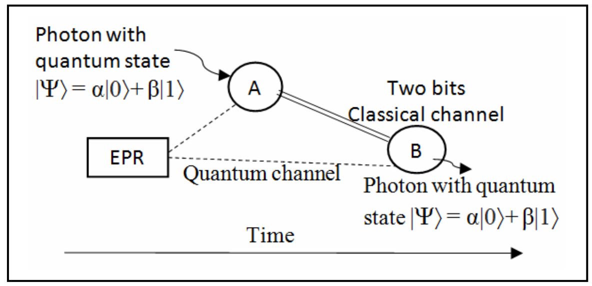

Figure 1. Basic Teleportation

The protocol is then as follows:

1. An EPR pair $|\Psi\rangle_0$ = $(|00" (" + |11" (")/(2$ is generated by a simple EPR circuit consisting of a Hadamard gate followed by a CNOT gate. The generated pair is shared by A and B. (i.e) one qubit sent to Alice, the other to Bob.

2. Now A wants to send the unknown quantum state $|\Psi\rangle = \alpha|0\rangle_{+}\beta|1\rangle$ to B. To do so, A uses the EPR circuit in reverse order, i.e., a CNOT followed by a Hadamard. The circuit entangles the unknown state with A's half of the EPR pair which is already entangled with B's qubit.

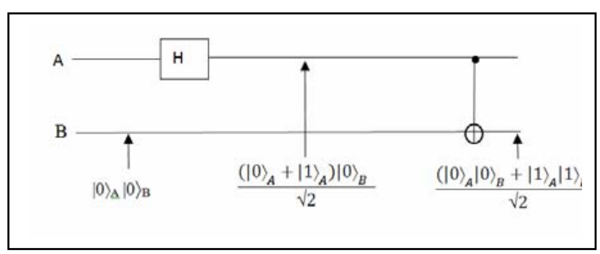

Figure 2. EPR pair generation using Hadamard and CNOT gates

3. At location A, a Bell measurement of the EPR pair qubit and the qubit to be teleported (the quantum state $|\Psi\rangle$) is performed, yielding one of four measurement outcomes, which can be encoded in two classical bits of information. Both qubits at location A are then discarded.

4. Using the classical channel, the two bits are sent from A to B.
5. As a result of the measurement performed at location A, the EPR pair qubit at location B is in one of four possible states. Of these four possible states, one is identical to the original quantum state $|\Psi\rangle$, and the other three are closely related. Which of these four possibilities actually obtains is encoded in the two classical bits. Knowing this, the qubit at location B is modified in one of three ways, or not at all, to result in a qubit identical to $|\Psi\rangle$, the qubit that was chosen for teleportation

Figure 3 shows the quantum teleportation circuit and its operation.

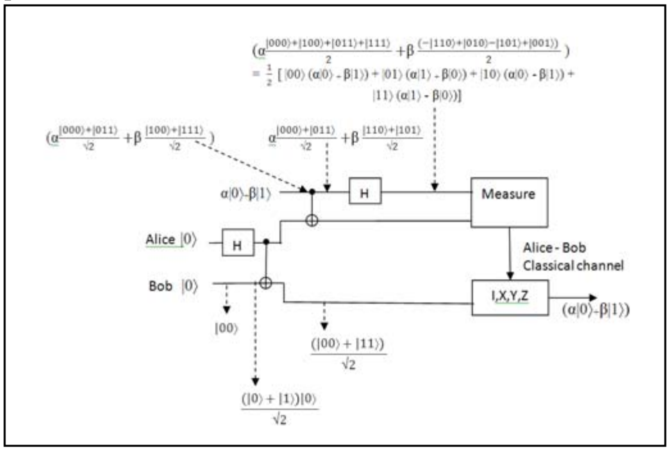

Figure 3.A Simple teleportation (Courtesy: Elements of Quantum Computing and Quantum Communication, Anirban Pathak) [3]

With the unknown state the initial state of the system is

$$|\Psi\rangle_1 = (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{|00\rangle+|11\rangle}{\sqrt{2}} = (\alpha|0\rangle \frac{|00\rangle+|11\rangle}{\sqrt{2}} + \beta|1\rangle \frac{|00\rangle+|11\rangle}{\sqrt{2}}) \quad (2)$$

Because of the measurement performed by Alice, the EPR pair qubit at location B is in one of four possible states. Table x. shows the Alice's measurement outcomes and corresponding unitary operations to be used by Bob to reproduce the unknown qubit state.

TABLE I. ALICE'S MEASUREMENT AND BOB'S REPRODUCTION

Alice Measures	State of Bob's Qubit	Bob's Operation	Bob's State after the operation
00	$(\alpha\|0\rangle + \beta\|1\rangle)$	I	$(\alpha\|0\rangle + \beta\|1\rangle)$
01	$(\alpha\|1\rangle + \beta\|0\rangle)$	X	$(\alpha\|0\rangle + \beta\|1\rangle)$
10	$(\alpha\|0\rangle - \beta\|1\rangle)$	Z	$(\alpha\|0\rangle + \beta\|1\rangle)$
11	$(\alpha\|1\rangle - \beta\|0\rangle)$	ZX= iY	$(\alpha\|0\rangle + \beta\|1\rangle)$

Using the first qubit of $|\Psi\rangle_1$ as the control qubit and second qubit as the target qubit the CNOT gate operation gives the state $|\Psi\rangle_2$ as

$$|\Psi\rangle_2 = (\alpha|0\rangle \frac{|00\rangle+|11\rangle}{\sqrt{2}} + \beta|1\rangle \frac{|10\rangle+|01\rangle}{\sqrt{2}}) \quad (3)$$

The Hadamard gate on the Alice path transforms the state of the system $|\Psi\rangle_2$ to $|\Psi\rangle_3$

$$|\Psi\rangle_3 = (\alpha \frac{(|0\rangle+|1\rangle)}{\sqrt{2}} \frac{(|00\rangle+|11\rangle)}{\sqrt{2}}) + \beta \frac{(|0\rangle-|1\rangle)}{\sqrt{2}} \frac{(|10\rangle+|01\rangle)}{\sqrt{2}})$$

$$= 1/2\, [\, |00\rangle (\alpha|0\rangle + \beta|1\rangle) + |01\rangle (\alpha|1\rangle + \beta|0\rangle) + |10\rangle (\alpha|0\rangle - \beta|1\rangle) + |11\rangle (\alpha|1\rangle - \beta|0\rangle)] \quad (4)$$

Alice measures the first two qubits and uses a classical channel to send the outcome of the measurement to Bob. Bob will be able to reconstruct the unknown state by applying appropriate Pauli gates on his qubit. This is so because Alice's measurement completely determines the state of the Bob's qubit and that allows Bob to choose an appropriate quantum gate which can reproduce the unknown quantum state $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$.Thus the unknown quantum state is teleported with the help of shared Bell state and two classical bits.

The simulation results are based on the QLIB - a library for quantum Information processing by Machnes[3] and are tabulated in Table 2.

III CONCLUSION

The physical realization of quantum teleportation which is very important in quantum cryptography has simulated and demonstrated using the qlibpakage for matlab.

REFERENCES

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[2] Ibrahim NQader and Ramzan Kog, Simulation of controlled quantum gates by using Mathematica.International Journal of Computer Science and Networks security, Vol.14.No. January 2014. Pp.59-64.

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[4] S. Machnes, quant-ph/0708.0478

TABLE II. ALICE'S MEASUREMENT AND BOB'S REPRODUCTION

Preparation		**Action on Alice Side**		**Measure on Alice Side**			**Bob Side**		
Shared_qubit pair $|\Psi\rangle_0$	***Overall state*** $|\Psi\rangle_1$	***CNOT (data,Alice)*** $|\Psi\rangle_2$	***Hadamard (data):*** $|\Psi\rangle_3$	***Probability***	***Measured bits***	***State at Alice***	***The state at Bob***	***Correction***	***Final State*** $|\Psi\rangle$
0 0.7071 -0.7071 0	0 0.3741 -0.3741 0 0 0.3041+ 0.5172i -0.3041- 0.5172i 0	0 0.3741 -0.3741 0 -0.3041 - 0.5172i 0 0 0.3041 + 0.5172i	-0.2151- 0.3657i 0.2646 -0.2646 0.2151+ 0.3657i 0.2151+ 0.3657i 0.2646 -0.2646 -0.2151- 0.3657i	0.25	00	1 0 0 0	0.8486 -0.2682+ 0.4561i	ACT with Y (Y=iZ*X)	0.5291 0.4301 + 0.7315i
					01	0 1 0 0	0.5291 -0.4301- 0.7315i	ACT with X	0.5291 0.4301 + 0.7315i
					10	0 0 1 0	0.8486 0.2682- 0.4561i	ACT with Z	0.5291 0.4301 + 0.7315i
					11	0 0 0 1	0.5291 0.4301+ 0.7315i	ACT with Y	0.5291 0.4301 + 0.7315i

Thus the unknown state teleported to Bob side is 0.5291 0.4301 + 0.7315